Online Appendix

A Analytical Results

A.1 Proof Lemma 1

Each firm i chooses next period capital $k_{i,t+1}$ one period in advance. Idiosyncratic productivity follows

$$\ln \varepsilon_{i,t+1} = \xi_{i,t} + \epsilon_{i,t+1}, \tag{23}$$

where $\xi_{i,t} \sim N(0, \sigma_\xi^2)$ is observed at time t; $\epsilon_{i,t+1} \sim N(0, \sigma_\epsilon^2)$ is realized at the start of $t+1$; and $\xi_{i,t}, \epsilon_{i,t}$ are independent across both time and firms and mutually independent. There is no aggregate shock.

The firm's objective at time t is to maximize its present discounted expected future profits by choosing an investment plan $\{i_{t+j}\}_{j=0}^{\infty}$:

$$\mathbb{E}_t\left(\sum_{j=0}^{\infty} \frac{1}{(1+r)^j}(\varepsilon_{i,t+j}k_{i,t+j}^{\alpha} - i_{i,t+j})\right) \quad \text{s.t.} \quad k_{i,t+j+1} = (1-\delta)k_{i,j+t} + i_{i,t+j}, \quad \text{given} \quad k_{i,t}, \tag{24}$$

where r is the interest rate faced by the firm and δ is the depreciation rate of capital.

The first-order condition for $k_{i,t+1}$ is

$$\alpha E_t(\varepsilon_{i,t+1})k_{i,t+1}^{\alpha-1} = r + \delta. \tag{25}$$

Solving (25) for $k_{i,t+1}$ and taking logs yields

$$\ln k_{i,t+1} = \frac{1}{1-\alpha}\ln E_t(\varepsilon_{i,t+1}) - \frac{1}{1-\alpha}\ln\left(\frac{r+\delta}{\alpha}\right). \tag{26}$$

Given the log-normal structure,

$$E_t(\varepsilon_{i,t+1}) = \exp\left(\xi_{i,t} + \tfrac{1}{2}\sigma_\epsilon^2\right). \tag{27}$$

Substituting into (26) gives

$$\ln k_{i,t+1} = A + B\xi_{i,t}, \qquad A := \frac{1}{1-\alpha}\Big(\tfrac{1}{2}\sigma_\epsilon^2 - \ln\frac{r+\delta}{\alpha}\Big), \quad B := \frac{1}{1-\alpha}. \tag{28}$$

Hence $\ln k_{i,t+1} \sim N(A, B^2\sigma_\xi^2)$ with

$$E(\ln k) = A, \qquad \mathrm{Var}(\ln k) = \frac{\sigma_\xi^2}{(1-\alpha)^2}. \tag{29}$$

We have $\ln\varepsilon = \xi + \epsilon$ and $\ln k = A + B\xi$. Because (ξ, ϵ) is jointly normal and $\ln k$ is affine in ξ, $(\ln\varepsilon, \ln k)$ is bivariate normal. Their covariance is

$$\mathrm{Cov}(\ln\varepsilon, \ln k) = B\mathrm{Var}(\xi) = \frac{\sigma_\xi^2}{1-\alpha}. \tag{30}$$

Output $y = \varepsilon k^\alpha$ implies

$$\ln y = \ln\varepsilon + \alpha\ln k. \tag{31}$$

Thus $\ln y$ is normal with mean

$$\mu_y := E(\ln y) = E(\ln\varepsilon) + \alpha E(\ln k) = \tfrac{1}{2}\sigma_\epsilon^2 + \alpha A, \tag{32}$$

and variance

$$\sigma_y^2 := \mathrm{Var}(\ln y) = \mathrm{Var}(\ln\varepsilon) + \alpha^2\mathrm{Var}(\ln k) + 2\alpha\mathrm{Cov}(\ln\varepsilon, \ln k) \tag{33}$$

$$= (\sigma_\xi^2 + \sigma_\epsilon^2) + \alpha^2\frac{\sigma_\xi^2}{(1-\alpha)^2} + 2\alpha\frac{\sigma_\xi^2}{1-\alpha} \tag{34}$$

$$= \sigma_\epsilon^2 + \frac{\sigma_\xi^2}{(1-\alpha)^2}. \tag{35}$$

Equations (32)–(35) complete the proof of Lemma 1. ■

A.2 Proof of Proposition 1

Let $Y = \sum_{i=1}^M y_i$ with y_i i.i.d. $\sim LN(\mu_y, \sigma_y^2)$ as in Lemma 1. Because the first two moments of a log-normal variable are available in closed form and the $\{y_i\}_{i=1}^M$ are independent across firms, we obtain $E(Y)$ and $\mathrm{Var}(Y)$ exactly by (i) linearity of expectation and (ii) the fact that all cross-firm covariances vanish.[26] We now report these exact moments.

For $y_i \sim LN(\mu_y, \sigma_y^2)$,

$$E(y_i) = e^{\mu_y + \sigma_y^2/2}, \qquad \mathrm{Var}(y_i) = (e^{\sigma_y^2} - 1)e^{2\mu_y + \sigma_y^2}. \tag{36}$$

Independence implies

$$E(Y) = M e^{\mu_y + \sigma_y^2/2}, \tag{37}$$

$$\mathrm{Var}(Y) = M(e^{\sigma_y^2} - 1)\, e^{2\mu_y + \sigma_y^2}. \tag{38}$$

Hence the (exact) coefficient of variation is

$$CV(Y) = \frac{\sqrt{\mathrm{Var}(Y)}}{E(Y)} = \sqrt{\frac{e^{\sigma_y^2} - 1}{M}}. \tag{39}$$

B Frictionless Economy

In this Appendix, we show that the endogenous aggregate μ can be exactly characterized by the first moment of the marginal distribution of capital K and the dynamic productivity distribution h with $\psi = 0$.

In equation (9), the choice of the current level of employment can be derived from a static problem as:

$$N(\varepsilon, k; \mu) = \arg\max_n [\varepsilon k^\alpha n^\nu - \omega(\mu) n] \tag{40}$$

which yields

$$N(\varepsilon, k; \mu) = [\nu \varepsilon k^\alpha / \omega(\mu)]^{1/(1-\nu)} \tag{41}$$

Using this decision rule for employment, we can replace the first and second terms in equation

[26] The sum of i.i.d. log-normal variables is not log-normal in general. A widely used approximation (Fenton–Wilkinson; see Marlow, 1967) replaces Y with $\widetilde{Y} \sim LN(\mu_Y, \sigma_Y^2)$ chosen so that $E(\widetilde{Y}) = E(Y)$ and $\mathrm{Var}(\widetilde{Y}) = \mathrm{Var}(Y)$. Solving gives $\sigma_Y^2 = \ln\big[(e^{\sigma_y^2} - 1)/M + 1\big]$ and $\mu_Y = \ln M + \mu_y + \sigma_y^2/2 - \sigma_Y^2/2$. Because the approximation matches moments by construction, (37)–(38) remain the exact values. The approximation is only needed if one wants a closed-form pdf/cdf for Y (e.g., tail probabilities).

(9) as:

$$\varepsilon k^{\alpha} n^{\nu} - \omega(\mu) n = (1-\nu)\varepsilon^{1/(1-\nu)} k^{\alpha/(1-\nu)} \left(\frac{\nu}{\omega(\mu)}\right)^{\nu/(1-\nu)}, \tag{42}$$

and we can rewrite the problem as follows:

$$\begin{aligned} v(\varepsilon, k; \mu) = \max_{k'} \Big[&(1-\nu)\varepsilon^{1/(1-\nu)} k^{\alpha/(1-\nu)} \left(\frac{\nu}{\omega(\mu)}\right)^{\nu/(1-\nu)} + (1-\delta)k - k' \\ &+ E\big[d(\mu, \mu') v(\varepsilon', k'; \mu') \mid \varepsilon, \mu\big]\Big]. \end{aligned} \tag{43}$$

This problem yields the optimal investment decision $G(\varepsilon; \mu)$ as follows:

$$G(\varepsilon; \mu) = L_0(\varepsilon) L_1(\mu) \tag{44}$$

$$L_0(\varepsilon; \mu) = \left(\sum_{\mu'} \Pi^{\mu}(\mu'|\mu) \sum_{\varepsilon'} \Pi^{\varepsilon}_{\mu'|\mu}(\varepsilon'|\varepsilon) \varepsilon'^{1/(1-\nu)} \right)^{(1-\nu)/(1-(\alpha+\nu))} \tag{45}$$

$$L_1(\mu) = \left(\frac{1 - (1-\delta)\sum_{\mu'} \Pi^{\mu}(\mu'|\mu) d(\mu, \mu')}{\alpha \sum_{\mu'} \Pi^{\mu}(\mu'|\mu) d(\mu, \mu') \left(\frac{\nu}{\omega(\mu')}\right)^{\nu/(1-\nu)}} \right)^{\frac{1-\nu}{\alpha+\nu-1}}. \tag{46}$$

This shows that the investment decision is independent of the current capital stock k, which depends only on the idiosyncratic productivity of the previous period. This implies that (1) it is sufficient to track the idiosyncratic productivity both in the current and previous periods for each firm and (2) the distribution of the current and previous idiosyncratic productivity h is a N_{ε} by N_{ε} grid point object.

It follows that the distribution of firms over idiosyncratic productivity and capital stock can be recovered, $\mu(\varepsilon_i, k_j)$, from h in each period as follows. First, we can construct $\mu_{\varepsilon,-1}(\varepsilon_j)$, the marginal distribution of firms over ε_j for $j = 1, \ldots, N_{\varepsilon}$ in the previous period -1, and $\mu_{\varepsilon}(\varepsilon_i)$, the marginal distribution of firms over ε_i for $i = 1, \ldots, N_{\varepsilon}$ in the current period. We can also construct $\Pi^{\varepsilon}_h(\varepsilon_j, \varepsilon_i)$, the transition probability $Pr(\varepsilon = \varepsilon_i \mid \varepsilon_{-1} = \varepsilon_j)$. Therefore, we can construct $\mu(\varepsilon, k)$, the distribution of firms over productivity and stock of capital in each period as

$$\mu(\varepsilon_i, k_j) = \mu_{\varepsilon,-1}(\varepsilon_j)\Pi_h^{\varepsilon}(\varepsilon_j, \varepsilon_i) \tag{47}$$

$$k_j = \frac{L_0(\varepsilon_j)\mu_{\varepsilon,-1}(\varepsilon_j)}{\sum_m L_0(\varepsilon_m)\mu_{\varepsilon,-1}(\varepsilon_m)} K \tag{48}$$

C Model Solution

In this section, we first outline the methodology used to construct the set of firm distributions across transitions and the corresponding probability space. We then describe the approach employed to solve the heterogeneous firm model.

C.1 Firm Distribution Across Transitions and its Probability Space

We construct the set of firm distributions across transitions, denoted by H, along with the associated transition probability $\Pi(h' \mid h)$, to replicate the fluctuations in the marginal distribution of firms over productivity, $\mu_{\varepsilon}(\varepsilon)$. To construct the transition matrix $\Pi(h' \mid h)$, we approximate the multivariate stochastic process governing the set of moments m^{ε} of the distribution $\mu_{\varepsilon}(\varepsilon)$ using a finite-state Markov chain. We then use this discretized process, together with the ergodic idiosyncratic transition probability Π^{ε}, to construct the firm distribution across transitions, H.

C.1.1 Moments Selection and Construction of the transition probability $\Pi(h'|h)$

To make the problem computationally feasible, we construct the set of firm distributions across transitions, denoted by H, along with the associated transition probability $\Pi(h' \mid h)$, to replicate the fluctuations in the most volatile moments of the marginal productivity distribution, $\mu_{\varepsilon}(\varepsilon)$. To this end, after simulating $\mu_{\varepsilon}(\varepsilon)$, we implement the following steps:

1. Identify the granular segment of the productivity distribution—specifically, the region of $\mu_{\varepsilon}(\varepsilon)$ where the Law of Large Numbers fails.

2. Select the set of most volatile moments that allow us to replicate the fluctuations in the firm distribution within this granular segment.

3. Approximate the stochastic process governing the selected moments m^{ε} of $\mu_{\varepsilon}(\varepsilon)$ using a multivariate first-order Markov chain, following the discretization method of Terry and Knotek II (2011). This yields a transition matrix $\Pi^{m^{\varepsilon}}(m^{\varepsilon\prime} \mid m^{\varepsilon})$.

Note that since ε follows a first-order Markov process, we have $\Pr(h' \mid h) \equiv \Pr(m^{\varepsilon\prime} \mid m^{\varepsilon})$. Therefore, we set $\Pi^{h} = \Pi^{m^{\varepsilon}}$.

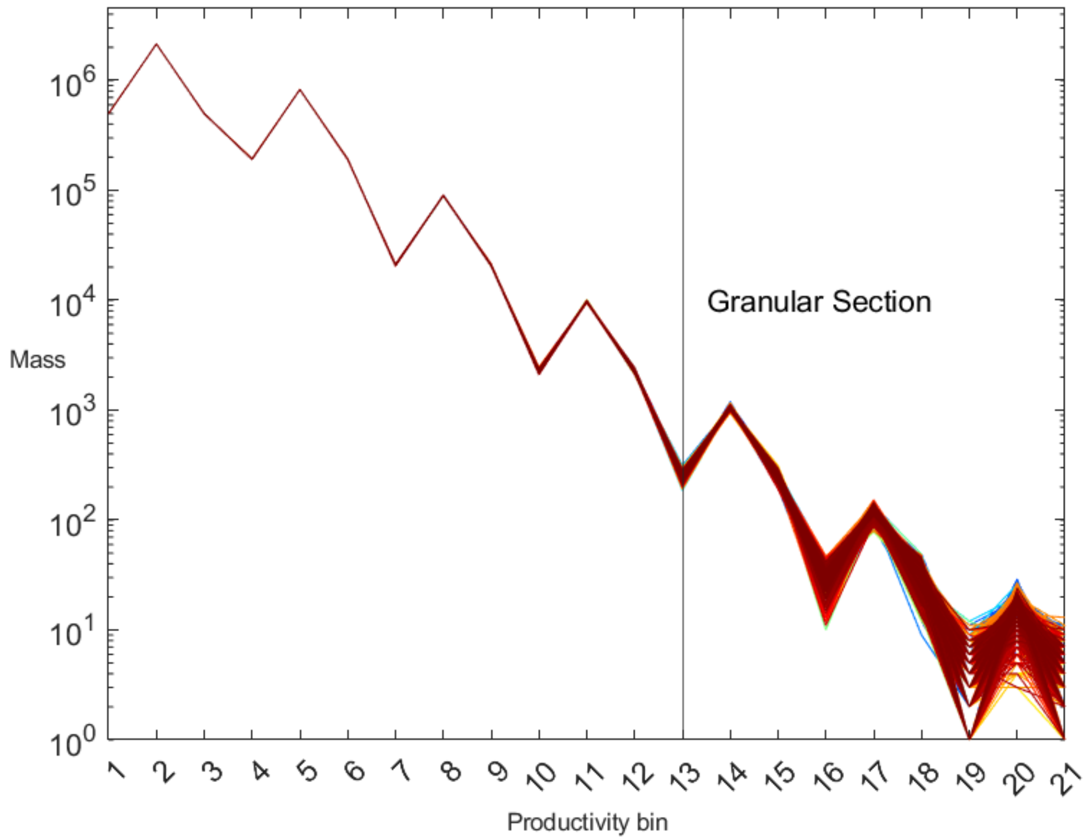
Figure 2 presents the simulated productivity distributions, $\mu_{\varepsilon}(\varepsilon)$, on a logarithmic scale over 10,000 periods. The simulation indicates that the Law of Large Numbers does not hold in the right tail of the productivity distribution. Consequently, we define the granular section of the productivity distribution as the portion of $\mu_{\varepsilon}(\varepsilon)$ associated with $\varepsilon \geqslant \varepsilon_{13}$.

Next, Figure 2 reports the distributions of the percentage deviations from their ergodic values for the moments that characterize fluctuations in the productivity distribution $\mu_{\varepsilon}(\varepsilon)$ within the granular section. Specifically, Figure 3 displays the percentage deviations of: (A) mean productivity, (B) the standard deviation of productivity, (C) Pearson's moment coefficient of skewness, and (D) firm mass—each calculated among firms with $\varepsilon \geqslant \varepsilon_{13}$.

Interestingly, the simulation reveals that the dispersion of the percentage deviation in the mean (0.398) is relatively low compared to those of the standard deviation (5.655), skewness (8.632), and firm mass (2.999). Based on this, we simulate a granular economy that replicates the cyclicality of: (1) the standard deviation, (2) Pearson's moment coefficient of skewness, and (3) firm mass within the granular section of the productivity distribution.

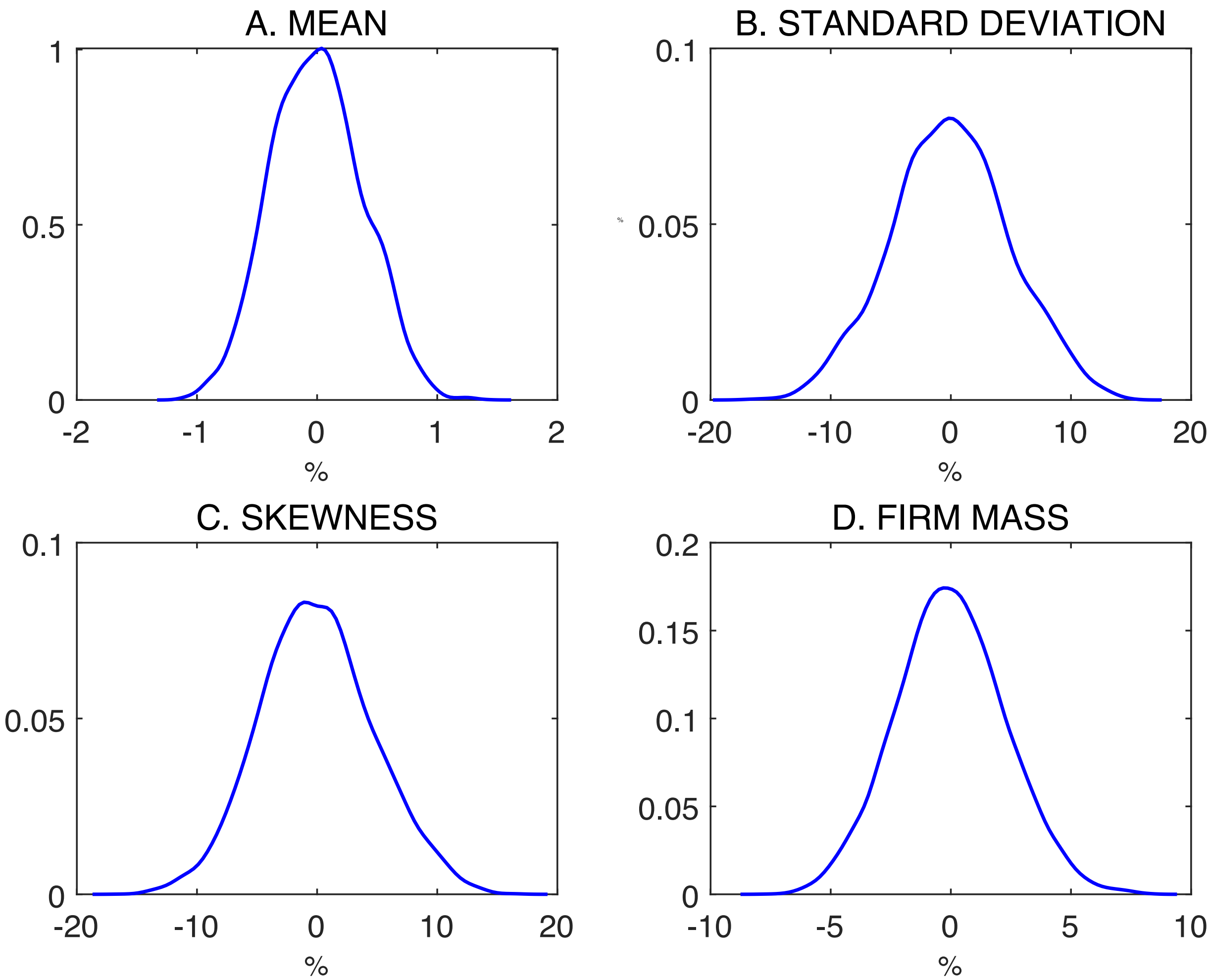
To discretize the multivariate stochastic process of the moment m^{ε}, we use a grid with three points for each moment, i.e., $d_{m^{\varepsilon}} = 3$.

Figure 2: Simulated Productivity Distribution $\mu_\varepsilon(\varepsilon)$



Notes: This figure illustrates the simulated productivity distribution, $\mu_\varepsilon(\varepsilon)$, on a logarithmic scale over 10,000 periods. Specifically, it depicts the mass of firms corresponding to each productivity level.

Figure 3: Moments of the Tail



Notes: The figure reports the distributions of percentage deviations from their ergodic values for: (A) mean productivity, (B) standard deviation of productivity, (C) Pearson's moment coefficient of skewness, and (D) firm mass—each calculated among firms with $\varepsilon \geqslant \varepsilon_{13}$. These distributions are derived from a 10,000-period simulation of the productivity distribution, $\mu_\varepsilon(\varepsilon)$.

C.1.2 Constructing H

We construct the set of firm distributions across transitions,

$$H = \{h_1, h_2, \ldots, h_{N_h-1}, h_{N_h}\},$$

by iterating steps 1 and 2 for F times, in order to ultimately implement step 3:

1. Construct

$$H^f = \{\mu^f_{\varepsilon,1}, \mu^f_{\varepsilon,2}, \ldots, \mu^f_{\varepsilon,N_h}\}$$

to reflect the discretized distribution m^ε.

2. Given Π^ε, implement a constrained draw of h such that each element of the set

$$H^f = \{h_1^f, h_2^f, \dots, h_{l+(m^\varepsilon-1)N_{\mu_\varepsilon}}^f, h_{l+(m^\varepsilon-1)N_{\mu_\varepsilon}+1}^f, \dots, h_{N_{\mu_\varepsilon}^2}^f\}$$

satisfies the following conditions for all $p, q = 1, 2, \dots, N_{\mu_\varepsilon}$ and $i, j = 1, 2, \dots, N_\varepsilon$:

$$\mu_{\varepsilon,q}^f(\varepsilon_i) = \sum_j h_{p+(q-1)N_{\mu_\varepsilon}}^f(\varepsilon_i, \varepsilon_j),$$
$$\mu_{\varepsilon,p}^f(\varepsilon_j) = \sum_i h_{p+(q-1)N_{\mu_\varepsilon}}^f(\varepsilon_i, \varepsilon_j),$$

where $N_{\mu_\varepsilon} = (\text{number of moments})^{d_{m^\varepsilon}}$.

3. Compute the average distribution:

$$H = \frac{1}{F}\sum_{f=1}^{F} H^f.$$

C.2 Model Simulation

C.2.1 Firm's Problem

We can rewrite the firm's problem (Equation 9) using marginal utility, $p \equiv U_c'(c, n^h)$, as follows:

$$\widehat{v}(\varepsilon, k; \mu) = \max_{n,i} p(\mu)\left[\varepsilon F(k, n) - \omega(\mu)n - i\right] + \beta \sum_{\mu'} \Pi^\mu(\mu'|\mu) \sum_{\varepsilon'} \Pi_{\mu'|\mu}^\varepsilon(\varepsilon'|\varepsilon)\widehat{v}(\varepsilon', k'; \mu') \quad (49)$$

The evolution of the aggregate equilibrium is characterized by the following mappings:

$$\begin{aligned} p &= \Gamma_p(\mu), \\ \mu' &\sim G(\mu). \end{aligned} \quad (50)$$

Given the transition probabilities defined above, we express the Bellman equation in terms of h and K, explicitly substituting these for μ:

$$v(\varepsilon, k; h, K) = \max_{n,i} p(h, K) \left[\varepsilon F(k, n) - \omega(h, K)n - i\right] + \beta \sum_{h'} \Pi^h(h'|h) \sum_{\varepsilon'} \Pi^{\varepsilon}_{h'}(\varepsilon'|\varepsilon) v(\varepsilon', k'; h', K') \tag{51}$$

We approximate the equilibrium mappings using log-linear rules:

$$\ln(\widehat{p}) = \alpha_p(h) + \beta_p(h) \ln(K), \tag{52}$$

$$\ln(\widehat{K}') = \alpha_K(h) + \beta_K(h) \ln(K) \tag{53}$$

These log-linear rules are used to forecast the future values of the price level p and aggregate capital stock K'. The coefficients α and β depend on the current aggregate state h.

C.2.2 Solution Algorithm

The solution algorithm consists of an inner and outer loop:

- **Inner Loop:** Solve the firm's value function, $v(\varepsilon, k; h, K)$, via value function iteration, holding the forecasting rules for aggregate variables (Equations 52) fixed.
- **Outer Loop:** Simulate the economy forward. In each period, draw a new aggregate state h' based on Π^h. Use the firm's policy functions (from the inner loop) and the transition probabilities $\Pi^{\varepsilon}_{h'}$ to determine firms' optimal decisions and update the aggregate capital stock and the distribution of firms. Then, update the forecasting rules.

The algorithm iterates between the inner and outer loops until the coefficients of the forecasting rules converge. This provides an approximate solution to the recursive equilibrium.

C.3 Performance

Our procedure to build firm distributions across transitions delivers an error of 0.0000830440, computed as follows:

$$\frac{1}{N_h}\sum_{j=1}^{N_h}\frac{1}{3}\left(\frac{\left|m_{1,j}^{\text{target}}-\text{st.dev.}(h_j)\right|}{m_{1,j}^{\text{target}}}+\frac{\left|m_{2,j}^{\text{target}}-\text{skew}(h_j)\right|}{m_{2,j}^{\text{target}}}+\frac{\left|m_{3,j}^{\text{target}}-\text{Tail Mass}(h_j)\right|}{m_{3,j}^{\text{target}}}\right) \quad (54)$$

Figure 4 presents the kernel density estimates of the R^2 values for the 138 forecasting rules applied to price and capital. For the capital forecasting rules, the mean and minimum R^2 values are 0.9992 and 0.9891, respectively. Similarly, for the price forecasting rules, the mean and minimum R^2 values are 0.9992 and 0.9962.

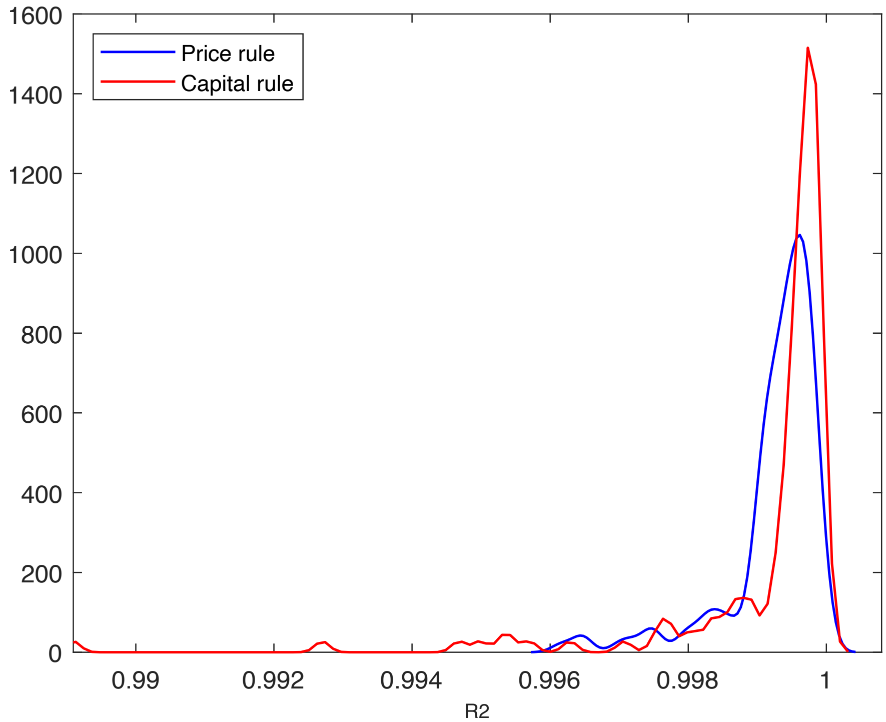


Figure 4: Distribution of the R^2 of the Forecasting Rules

Notes: The figure reports the kernel densities of the R^2 values for the 138 forecasting rules. The blue line corresponds to price forecasting rules, and the red line to capital forecasting rules.

D Data

Sample Selection.— We construct an annual panel dataset by merging firm-level TFP estimates from İmrohoroğlu and Tüzel (2014) with Compustat data from 1964 to 2019. Using Standard Industry Classification (SIC) codes, we exclude firms in the oil, energy, and financial sectors from our sample. Specifically, we exclude:

- **Oil and oil-related firms:** SIC codes 2911, 5172, 1311, 4922, 4923, 4924, and 1389;
- **Energy firms:** SIC codes ranging from 4900 to 4940;
- **Financial firms:** SIC codes ranging from 6000 to 6999.

We also eliminate firms with missing data to ensure valid sales observations across the sample. The TFP estimates are adjusted using industry-specific time dummies to remove industry-year effects.

Variable Construction.—We define the variables used in our empirical analysis as follows:

1. *Gross investment rate:* Ratio of capital expenditures (CAPX) to plant, property, and equipment (PPEGT). We control for 2-digit sector-by-year fixed effects.

2. *Inaction rate:* Mass of firms such that $|i/k| \leqslant 0.01$.

3. *Net investment rate:* Growth rate of PPEGT. We control for 2-digit sector-by-year fixed effects.

4. *Negative investment spike:* Among the 500 largest firms (by sales) in the previous period, defined as the mass of firms such that

$$\Pr\left(\frac{\Delta k'}{k} \leqslant -0.2\right).$$

5. *Marginal product of capital* (MPK): Logarithm of the ratio of sales (SALE) to physical capital (PPEGT). We control for 2-digit sector-by-year fixed effects.

6. *Idiosyncratic shocks:* Approximated by the productivity growth rate:

$$\Delta\varepsilon_{i,t} = \frac{\varepsilon_{i,t} - \varepsilon_{i,t-1}}{\varepsilon_{i,t-1}}. \tag{55}$$

7. *Granular residual:* Difference between the sales-weighted and equally weighted average of idiosyncratic shocks among the 200 largest firms in the previous period:

$$\Theta_t^{\text{Gabaix}} = \sum_{i=1}^{100} \frac{\text{Sale}_{i,t-1}}{\text{Sale}_{t-1}^{100}} \Delta\varepsilon_{i,t} - \frac{1}{100} \sum_{i=1}^{100} \Delta\varepsilon_{i,t}. \tag{56}$$

8. *Granular capital misallocation:* Dispersion of MPK among the 200 largest firms:

$$\sigma_t^{\text{MPK,200}} = \sqrt{\frac{1}{200} \sum_{i=1}^{200} (MPK_{i,t} - MPK_t^{\text{mean}})^2}. \tag{57}$$

Table 6: Summary Statistics

x	Observations	mean	sd	min	max
Θ_t^{Gabaix}	38	0.000	0.017	-0.031	0.056
$\ln(\sigma^{MPK,200})$	39	0.000	0.026	-0.055	0.0771
$\ln(\sigma^{MPK,-200})$	39	0.000	0.017	-0.048	0.047
$Pr(\lvert\frac{i}{k}\rvert \leqslant 0.01)$ TOP 500	39	0.111	0.021	0.075	0.186
$Pr(\frac{\Delta k'}{k} \leqslant -0.2)$ TOP 500	39	0.024	0.0282	0.000	0.197

Notes: This table reports descriptive statistics for the variables used in Section 3. All series are HP-filtered with a smoothing parameter of 6.25, except for the inaction rate and the negative investment spike.

E Equilibrium Wage Dynamics and Business Cycles

Table 7: The Impact of Frictions and General Equilibrium on Business Cycles

Variable	**Statistic**	**Benchmark (GE)** ($\psi = 0.35$)	**Frictionless (GE)** ($\psi = 0.00$)	**Partial Equilibrium** **(Frictionless)**
Output (Y)	$\sigma(x)$	0.294	0.377	0.858
	$\sigma(x)/\sigma(Y)$	1.000	1.000	1.000
	$\rho(x, Y)$	1.000	1.000	1.000
Consumption (C)	$\sigma(x)$	0.171	0.119	2.702
	$\sigma(x)/\sigma(Y)$	0.582	0.316	3.150
	$\rho(x, Y)$	0.786	0.716	0.442
Investment (I)	$\sigma(x)$	1.200	1.962	14.706
	$\sigma(x)/\sigma(Y)$	4.088	5.209	17.146
	$\rho(x, Y)$	0.901	0.975	0.050
Hours (H)	$\sigma(x)$	0.191	0.303	0.858
	$\sigma(x)/\sigma(Y)$	0.651	0.805	1.000
	$\rho(x, Y)$	0.834	0.962	1.000

Notes: This table compares business cycle moments across three model specifications to isolate the effects of capital frictions and general equilibrium wage adjustments. (a) Column 1 reports the benchmark model with capital irreversibility ($\psi = 0.35$); (b) Column 2 reports the general equilibrium model without capital irreversibility ($\psi = 0.00$); (c) Column 3 reports a frictionless model in partial equilibrium, where wages do not adjust to clear the aggregate labor market.

F Cyclicality of the Real Interest Rate over Time

Table 8: Cyclical Dynamics of Risk-Free Rate

	$\sigma(\ln(1+r))$	$\rho(\ln(1+r), \ln Y)$
Whole sample	1.310	-0.286
Pre-1980	1.131	-0.828
Post-1980	1.178	0.189

Notes: Real interest rate is measured as the nominal return on 1-year Treasury bills (DGS1 taken from FRED) adjusted for realized CPI inflation (CPIAUCSL_PC1 taken from FRED). Output measured as real gross domestic product (GDPCA taken from FRED). All series are HP-filtered in logs with a smoothing parameter of 6.25, following Ravn and Uhlig (2002). Whole sample refers to the 1964–2019 series. Pre-1980 refers to the 1964:1979 sample. Post-1980 refers to the 1980–2019 sample.

G Decomposition of Irreversibility's Impact

Table 9: Decomposition on the Effect of Capital Irreversibility and Misallocation

	Benchmark		*No T.V. Depreciation*		*Household Renting Capital*	
	$\frac{\sigma(x)}{\sigma(x^{\text{No-irrev.}})}$	$\rho(x, \ln Y)$	$\frac{\sigma(x)}{\sigma(x^{\text{No-irrev.}})}$	$\rho(x, \ln Y)$	$\frac{\sigma(x)}{\sigma(x^{\text{No-irrev.}})}$	$\rho(x, \ln Y)$
$\ln Y$	0.780	1.000	0.947	1.000	0.499	1.000
$\ln C$	1.437	0.786	1.050	0.702	0.882	0.702
$\ln I$	0.612	0.901	0.978	0.967	0.386	0.967
$\ln H$	0.629	0.834	0.936	0.950	0.383	0.950
$\ln(1 + r)$	1.443	-0.496	1.110	-0.148	0.889	0.823

Notes: This table compares the business cycle moments of the benchmark model, the model with only the *time-varying misallocation component*, and the model without both the *time-varying misallocation* and *time-varying depreciation* components. $\sigma(x)$ denotes the percentage standard deviation of variable x; $\sigma(x)/\sigma(x^{\text{No-irrev.}})$ is the relative standard deviation with respect to its counterpart in the model with $\psi = 0$; and $\rho(x, \ln Y)$ is the contemporaneous correlation of x with output Y. All series are HP-filtered with a smoothing parameter of 6.25, following Ravn and Uhlig (2002).

In Section 4.2, we discussed how investment irreversibility mitigates output volatility by distinguishing between its direct effects on firm-level decision-making and its broader general equilibrium implications. To assess the relative contribution of these two channels to aggregate volatility, we consider a counterfactual scenario: an economy with *no time-varying capital depreciation*, where capital adjustment costs are purely virtual. In this setting, while adjustment costs continue to influence firms' optimal capital choices, they do not entail any actual reallocation of real resources. Consequently, this setup partially removes the indirect general equilibrium effect, as the absence of real adjustment costs enhances the representative household's capacity to smooth consumption over time.[27]

[27]The indirect general equilibrium effect may still influence volatility in the *no time-varying capital depreciation* economy. Specifically, firms that remain within the inaction region and retain excessively large

Table 9 presents the relative volatilities of the *Benchmark* and *No Time-Varying Depreciation* economies, expressed relative to the *Frictionless* economy. The findings suggest that the indirect general equilibrium effect accounts for at least three-quarters of the total reduction in aggregate volatility.

We conclude by quantifying the contribution of capital misallocation fluctuations to granular volatility by analyzing a third counterfactual scenario: the *Household Renting Capital* economy. In this setting, capital is rented by the representative household to firms each period, ensuring that the marginal product of capital is equalized across firms and eliminating factor misallocation. As shown in Table 9, while irreversibility partially dampens the aggregate volatility arising from factor misallocation, it still accounts for more than one-third of granular volatility.

capital stocks reduce the representative household's capacity to smooth consumption over time.

H Aggregate TFP Shock Economy

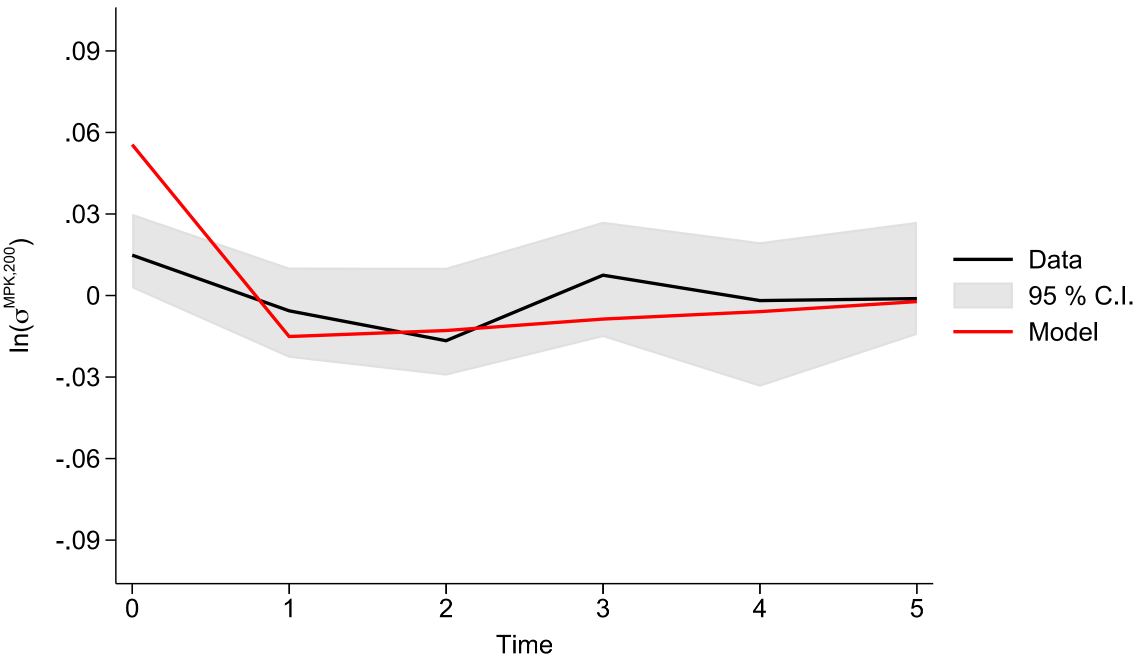
Table 10: Business Cycle Moments: Aggregate TFP Shock

	$\psi = 0.35$ with $y = Z\varepsilon k^{\alpha} l^{\nu}$			$\psi = 0.00$ with $y = Z\varepsilon k^{\alpha} l^{\nu}$		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, \ln Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, \ln Y)$
$\ln Y$	1.888	1.000	1.000	1.898	1.000	1.000
$\ln C$	0.981	0.520	0.924	0.957	0.504	0.934
$\ln I$	6.775	3.589	0.963	7.412	3.905	0.965
$\ln H$	1.051	0.557	0.934	1.062	0.559	0.946
$\ln(1+r)$	0.169	0.089	0.848	0.151	0.079	0.871

Notes: The table compares the business cycle moments of model economies whose volatility is exclusively driven by aggregate TFP shock. In particular, $\psi > 0$ refers to the economy with partial capital irreversibility, and $\psi = 0$ refers to the economy without partial irreversibility. $\sigma(x)$ is the percentage standard deviation of x, and $\sigma(x)/\sigma(\ln Y)$ is the relative standard deviation to that of Y, and $\rho(x, \ln Y)$ is the contemporaneous correlation of x with Y. All series are HP-filtered with a smoothing parameter of 6.25, following Ravn and Uhlig (2002).

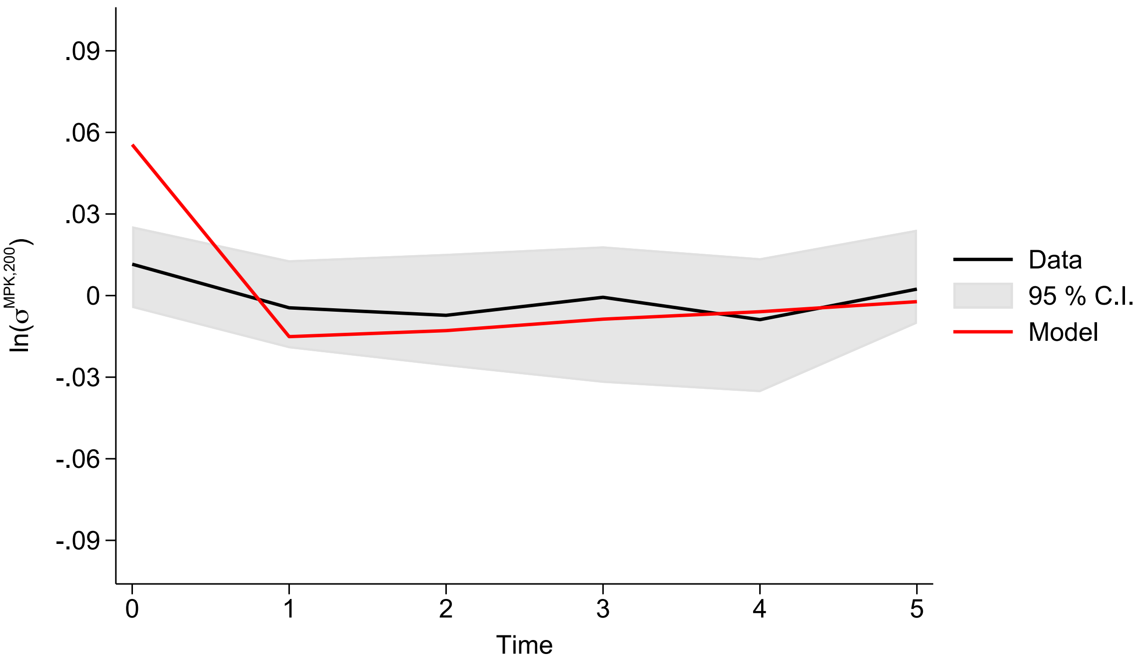
I Granular shocks and Misallocation: Robustness Checks

Figure 5: Effect of Granular Residual Shock on Granular Misallocation



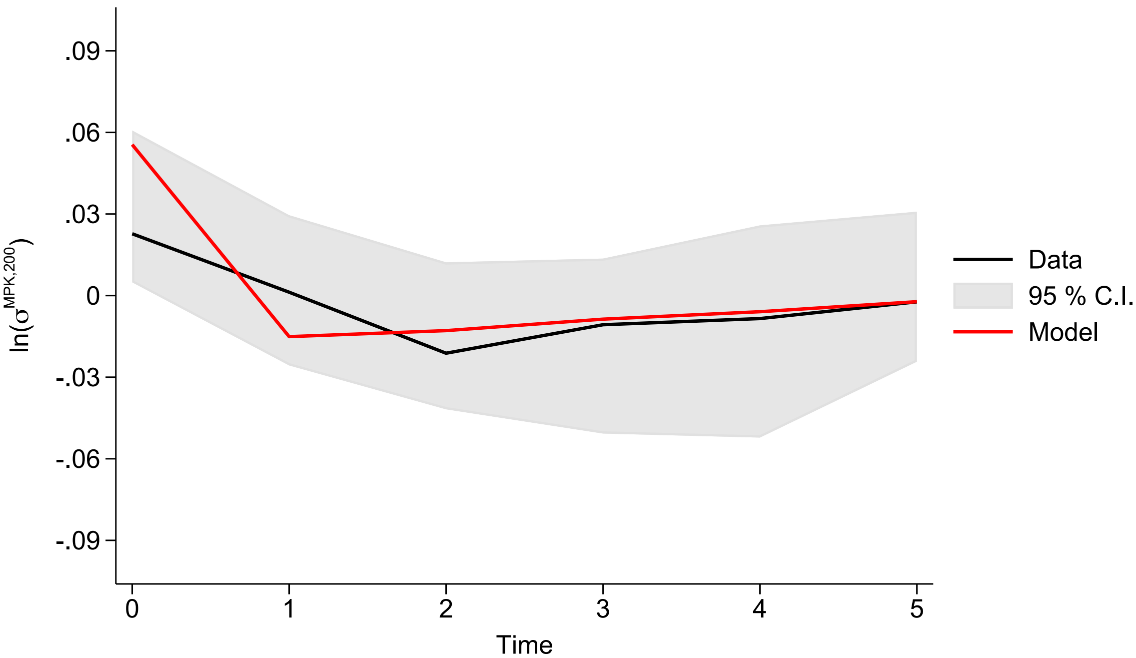
Notes: Responses of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular residual shock. The black line represents the response estimated from the data. The red line represents the response obtained from the simulated data. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 1000 repetitions. All series are HP-filtered using a smoothing parameter of 6.25, following Ravn and Uhlig (2002). The sample period spans from 1980 to 2019. Granular misallocation is estimated conditional on 2-digit sector-by-year fixed effects. For further details on data construction, see Appendix D.

Figure 6: Effect of Granular Residual Shock on Granular Misallocation



Notes: Responses of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular residual shock. The black line represents the response estimated from the data. The red line represents the response obtained from the simulated data. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 1000 repetitions. All series are HP-filtered using a smoothing parameter of 6.25, following Ravn and Uhlig (2002). The sample period spans from 1964 to 2019. Granular misallocation is estimated conditional on 3-digit sector-by-year fixed effects. For further details on data construction, see Appendix D.

Figure 7: Effect of Granular Residual Shock on Granular Misallocation



Notes: Responses of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular residual shock. The black line represents the response estimated from the data. The red line represents the response obtained from the simulated data. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 1000 repetitions. All series are HP-filtered using a smoothing parameter of 6.25, following Ravn and Uhlig (2002). The sample refers to the period spanning from 1980 to 2019. Granular misallocation is estimated conditional on 3-digit sector-by-year fixed effects. For further details on data construction, see Appendix D.

J Policy Implications

Table 11: Business Cycle Moments

	Benchmark			*Subsidy*			*No-Price Gap*		
	$\exp(x)$	$\sigma(x)$	$\rho(x, \ln Y)$	$\exp(x)$	$\sigma(x)$	$\rho(x, \ln Y)$	$\exp(x)$	$\sigma(x)$	$\rho(x, \ln Y)$
$\ln Y$	0.778	0.295	1.000	0.802	0.320	1.000	0.787	0.377	1.000
$\ln C$	0.646	0.170	0.787	0.656	0.161	0.796	0.659	0.119	0.716
$\ln I$	0.132	1.210	0.905	0.131	1.263	0.862	0.128	1.962	0.975
$\ln H$	0.336	0.193	0.839	0.341	0.215	0.892	0.333	0.303	0.962
$\ln(1+r)$	1.042	0.025	-0.496	1.042	0.023	-0.332	1.042	0.018	-0.022

Notes: The table reports the business cycle moments of the *Benchmark* economy and the economies under the two downsize policies. $\sigma(x)$ is the percentage standard deviation of x, and $\rho(x, \ln Y)$ is the contemporaneous correlation of x with $\ln Y$. The model moments are obtained from a 15,000-period unconditional simulation using the solution of the model. The reported standard deviations and correlations refer to HP-filtered series in logarithms, using a smoothing parameter of 6.25, following Ravn and Uhlig (2002).