

Investment Irreversibility in a Granular World*

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Abstract

Investment irreversibility plays a prominent role in reducing fluctuations in both aggregate output and investment when aggregate uncertainty stems from idiosyncratic shocks to large firms—rather than aggregate shocks, under which irreversibility has little effect on volatility. The key is that idiosyncratic shocks are sufficiently volatile to cause the irreversibility constraint to bind cyclically for a significant mass of firms. Consequently, irreversibility hampers productivity-enhancing capital reallocation and impedes household consumption smoothing, increasing real wage volatility and dampening aggregate fluctuations. This stabilizing role, achieved by deterring disinvestment and limiting the depth of recessions, has implications for policy design: merely subsidizing capital liquidation yields only modest welfare gains. This is because such policies lead to excessive disinvestment; firms, compensated by the government, do not internalize the aggregate resource losses, thereby destabilizing the economy and offsetting the gains from smoother capital reallocation.

Keywords. Investment irreversibility; Business cycles; Idiosyncratic shocks; Capital misallocation

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1 Introduction

Investment irreversibility is a fundamental friction in capital allocation,¹ the substantial losses firms incur when liquidating equipment and machinery hamper efficient capital reallocation across firms.² This friction can be acute during recessions, when many firms wish to downsize but face liquidation discounts. Policymakers have long viewed this friction as grounds for intervention: the 2009 GM and Chrysler restructurings, for example, catalyzed bankruptcy reforms designed to facilitate capital transfers from distressed to productive firms while preserving asset values.³

Despite such policy relevance during recessions, existing quantitative analyses typically find negligible or modest aggregate effects of investment irreversibility (Sargent, 1980; Dow and Olson, 1992; Veracierto, 2002; Lanteri, 2018), providing limited insight into its cyclical role. Yet theory suggests irreversibility may matter. Irreversibility prevents efficient capital reallocation, worsening and prolonging downturns—suggesting that policies easing liquidation constraints could be welfare-enhancing. However, irreversibility may also limit recession depth by deterring excessive disinvestment, implying that such policies would remove this stabilizing force and deepen recessions.

In light of this ambiguity, this paper revisits the aggregate implications of investment irreversibility by posing a new question: does investment irreversibility affect business cycle fluctuations when the source of aggregate fluctuations is idiosyncratic shocks rather than

¹Seminal works on investment irreversibility include Arrow (1968), Nickell (1977), Pindyck (1988, 1991), Dixit and Pindyck (1994), Demers (1991), Leahy (1993), Abel and Eberly (1994, 1996), Bertola and Caballero (1994), Bertola (1998), Bloom (2009).

²Investment irreversibility can arise from multiple sources including asset specificity (Ramey and Shapiro, 2001; Lanteri, 2018), technological obsolescence (Caunedo and Keller, 2021), and thin resale markets (Ottonezzo, 2024). Evidence from systematic analysis of corporate liquidations reveals that even general-purpose equipment and machinery incur substantial losses upon disposal, typically recovering only 50-70% of replacement cost, while specialized assets face even higher losses of 10-30%. See Kermani and Ma (2023) for detailed analysis.

³Section 363 sales in bankruptcy law allow expedited asset transfers to preserve going-concern value and avoid piecemeal liquidation, which enabled rapid transfer of viable assets to “New GM” and the Fiat consortium, respectively, preserving operational continuity that would have been lost in traditional liquidation proceedings (Brubaker and Tabb, 2010; Lubben, 2009; Roe and Skeel, 2010).

economy-wide aggregate shocks? If so, how and how much? We answer these questions in a model of a “granular” economy, drawing on the recent argument that the economy can experience significant fluctuations originating from firm-level idiosyncratic shocks to large firms that do not average out.⁴ We find that investment irreversibility substantially dampens the volatility of aggregate output, investment, and hours worked, while amplifying consumption volatility. This occurs through two channels: a direct constraint on capital allocation that leads to marginal products of capital being unequalized across firms, and an indirect general equilibrium effect where impeded consumption smoothing leads to greater real wage flexibility, which acts as a shock absorber for firms.

A direct channel stems from the impact of irreversibility on capital allocation, a mechanism we initially illustrate using a simplified partial equilibrium model and then demonstrate within our fully fledged general equilibrium model. The partial equilibrium analysis reveals that aggregate volatility is tied to the covariance between firm-level productivity and capital; a lower covariance dampens economic fluctuations. Our quantitative general equilibrium model confirms that this is a key channel. In that setting, irreversibility causes firms’ capital stocks to adjust more sluggishly to idiosyncratic shocks to productivity at the firm level. This sluggishness reduces the dynamic alignment between capital and productivity, and as a consequence, aggregate investment becomes less volatile and less responsive to shocks.

An indirect general equilibrium channel works through wage flexibility, an effect absent from our partial equilibrium analysis. In the full general equilibrium model, when capital adjustment is restricted by irreversibility, households have a diminished ability to smooth consumption over time. This results in greater volatility in consumption, which in turn translates into more volatile movements in the real wage. This acts as a shock absorber for firms; a larger wage reduction during downturns helps mitigate the fall in employment and production. This effect contributes to lower volatility in aggregate hours worked and further dampens overall output fluctuations.

⁴See Gabaix (2011) for the original hypothesis.

This result stands in sharp contrast to the “irrelevance” result found in standard business cycle models that rely exclusively on aggregate shocks.⁵ The reason for this lies in the micro-level investment dynamics. We show that granular shocks, unlike their aggregate counterparts, are sufficiently varied and volatile to cause the irreversibility constraint to bind *cyclically* for a significant mass of economically important firms. Specifically, in our granular economy, the share of large firms wanting to downsize but being prevented by the constraint is not only substantial but also strongly counter-cyclical. This is a critical distinction from existing models with aggregate shocks, where the fraction of constrained large firms shows virtually no cyclical variation. This active, time-varying engagement of the constraint among large firms is what elevates investment irreversibility to a prominent propagation mechanism in a granular world.

The paper takes three steps to demonstrate the aforementioned result. We first sketch a simple model to study factors that determine the volatility of aggregate output when the economy is populated by a finite number of firms: that is, the economy is granular and idiosyncratic shocks may not cancel out in the cross-section due to the fact that the law of large numbers does not hold. We then build the above-mentioned fully fledged equilibrium business cycle model to quantitatively evaluate the relative contribution of capital misallocation in driving aggregate fluctuations. Finally, to validate our model predictions, we build a firm-level panel dataset by merging firm-level accounting information from Compustat with TFP estimations of İmrohoroglu and Tüzel (2014). We show that the model performs well in replicating the aggregate dynamics following a shock to the largest firms of the economy and the dynamics of capital misallocation among the largest firms of the economy.

Our analysis reveals that investment irreversibility in granular economies presents policy-

⁵In particular, investment irreversibility is often found to play a limited role for aggregate fluctuations in standard models driven by common, economy-wide shocks. One reason is that aggregate fluctuations driven by economy-wide shocks may not be sufficiently volatile or varied to cause the irreversibility constraint to bind cyclically for a significant mass of firms (Veracierto, 2002). Other research also suggests that lumpy investment arising from nonconvex adjustment costs plays a limited or insignificant role for aggregate dynamics. In such models, general equilibrium price adjustments, such as changes in wages and interest rates, can smooth out potential aggregate effects, leading to the same aggregate dynamics as in frictionless models (Thomas, 2002; Khan and Thomas, 2003, 2008).

makers with a trade-off: while it dampens aggregate volatility, it impairs capital allocation and consumption smoothing. We consider two counterfactual policy experiments: (1) subsidizing liquidation losses incurred by firms, and (2) eliminating frictions to enable capital reallocation without losses. The second policy eliminates irreversibility entirely and generates 2.52% welfare gains in consumption-equivalent terms. While evidently first-best and admittedly unrealistic, it serves as a benchmark to highlight how ineffective subsidizing liquidation losses proves to be—yielding only 0.34% gains. While the aim of the first policy is to free resources for more productive uses, it creates a perverse incentive: firms do not internalize the deadweight loss from capital destruction because they are compensated for liquidation losses. This leads to excessive disinvestment, generating resource loss in the economy and explaining why this policy yields only modest welfare gains.⁶

A key challenge in computing the equilibrium of the model under aggregate uncertainty (e.g. Khan and Thomas (2008) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018)) is constructing a feasible set of transition probabilities and their probability space that are consistent with the granularity of the underlying idiosyncratic productivity process. To address this, we employ the discretization method for multivariate stochastic processes developed by Terry and Knotek II (2011). This method allows us to simulate changes in transition probabilities that replicate the fluctuations of the moments of firm distribution over present and past productivities. This new approach enables the study of other real or financial frictions in economies characterized by granularity.

Related literature

This paper is related to several strands of literature. Firstly, this paper extends research on how firms' idiosyncratic shocks generate aggregate fluctuations (Jovanovic, 1987; Gabaix,

⁶Our findings provide a cautionary tale for capacity reduction policies implemented across many countries. Examples include Japan's shipbuilding capacity reduction under the 1978 Structurally Depressed Industries law, which paid for permanent scrapping of shipyards (United States General Accounting Office, 1983), and the UK's coal mine closure programs that provided redundancy payments to accelerate pit shutdowns (Glyn, 1992). While these policies aimed to free resources for more productive uses, they may have overstated their benefits by failing to account for the deadweight loss from capital destruction.

2011; Gabaix and Kojen, 2024; Carvalho and Grassi, 2019; Di Giovanni and Levchenko, 2012; Di Giovanni, Levchenko, and Méjean, 2014, 2024; Yeh, 2023; Ifergane, 2024; Burstein, Carvalho, and Grassi, 2025). This paper is also related to recent empirical work documenting fat-tailed and skewed firm dynamics (Salgado, Guvenen, and Bloom, 2019; Melcangi and Sarpietro, 2025; Ehouarne, Kuehn, and Schreindorfer, 2022). A distinct feature of our study is the inclusion of real frictions and capital misallocation—elements typically abstracted from in granular models—to understand how these documented micro-level shock patterns translate into aggregate fluctuations through the lens of investment irreversibility.

Secondly, we contribute to the literature on micro-level investment frictions and aggregate fluctuations, particularly to the irreversibility literature (e.g., Abel and Eberly (1994), Bertola and Caballero (1994), Bertola (1998), Pindyck (1988), Dixit and Pindyck (1994)).⁷ While Veracierto (2002) finds investment irreversibility largely irrelevant in economies driven by aggregate shocks, we demonstrate its crucial role when idiosyncratic shocks are the primary driver of economic fluctuations.

Thirdly, this paper is connected to the misallocation literature (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2010; David and Venkateswaran, 2019) and especially we advance research on capital misallocation in business cycles (Lanteri, 2018). Our novel approach examines how investment irreversibility in a granular economy affects capital misallocation dynamics, revealing how firm-level decisions can amplify idiosyncratic shocks and contribute to aggregate volatility.

Finally, our work complements research on the aggregate implications of higher-order moment shocks at the firm level (Bloom, 2009; Bloom et al., 2018; Salgado et al., 2019; Jaimovich, Terry, and Vincent, 2023).

⁷For example, see Caballero and Engel (1999), Thomas (2002), Khan and Thomas (2008), Bachmann and Bayer (2014), Bloom (2009), Winberry (2021), Blanco and Baley (2024) for more broader contexts of investment and capital adjustment frictions.

Organization

The rest of the paper is organized as follows. Section 2 introduces the granular shock mechanism in a partial equilibrium model. Section 3 develops the general equilibrium model and its solution method. Section 4 presents the model’s calibration and quantitative results. Section 5 provides empirical support using firm-level data. Section 6 discusses policy implications and concludes.

2 Analytical Insights into Capital Allocation and Aggregate Volatility

This section presents analytical results derived from a simple, partial equilibrium model to illustrate key mechanisms behind aggregate fluctuations in an economy with a finite number of firms. The primary aim is to understand the relationship between capital allocation and aggregate fluctuations when idiosyncratic shocks do not average out. While this particular model itself does not incorporate investment irreversibility to render analytical solutions feasible, we will use its framework to discuss how investment irreversibility, by potentially affecting capital allocation and the covariance between productivity and capital, *could* influence aggregate fluctuations. The purpose is to build intuition for these mechanisms, which are explored in a general equilibrium settings later.

We consider an economy populated by $M \in (0, \infty)$ firms. Each firm is indexed by i . Firms produce a homogeneous good in a perfectly competitive environment. Each firm uses capital as the sole input to produce, employing a decreasing returns to scale Cobb-Douglas production function:

$$y_i = \varepsilon_i k_i^\alpha \tag{1}$$

where k_i is the capital stock of firm i , and ε_i is its idiosyncratic productivity. We assume

that idiosyncratic productivity follows the stochastic process:

$$\ln(\varepsilon_{i,t+1}) = \xi_{i,t} + \epsilon_{i,t+1} \quad (2)$$

where $\xi_{i,t} \sim \mathcal{N}(0, \sigma_\xi)$ is a idiosyncratic component known at time t , and $\epsilon_{i,t+1} \sim \mathcal{N}(0, \sigma_\epsilon)$ is a component that is revealed at the beginning of period $t + 1$.

The firm's objective at time t is to maximize its present discounted expected future profits by choosing an investment plan $\{i_{i,t+j}\}_{j=0}^\infty$:

$$\mathbb{E}_t \left(\sum_{j=0}^{\infty} \frac{1}{(1+r)^j} (\varepsilon_{i,t+j} k_{i,t+j}^\alpha - i_{i,t+j}) \right) \quad \text{s.t.} \quad k_{i,t+j+1} = (1-\delta)k_{i,t+j} + i_{i,t+j}, \quad \text{given} \quad k_{i,t}. \quad (3)$$

where r is the interest rate faced by the firm (assumed to be constant for simplicity in this analytical section) and δ is the depreciation rate of capital.

Lemma 1. If firms choose the stock of capital one period ahead, then the idiosyncratic output y_i is log-normally distributed, $\ln(y_i) \sim \mathcal{N}(\mu_y, \sigma_y)$. The parameters of this distribution are:

$$\mu_y = \mathbb{E}(\ln(\varepsilon)) + \alpha \mathbb{E}(\ln(k)) \quad (4)$$

$$\sigma_y^2 = \mathbb{V}(\ln(\varepsilon)) + \alpha \left(\frac{2-\alpha}{1-\alpha} \right) \text{Cov}(\ln(\varepsilon), \ln(k)) \quad (5)$$

where μ_y represents the expected value of the log of the product εk^α , while σ_y^2 represents its variance. The distribution of $\ln(k)$ and the covariance between $\ln(k)$ and $\ln(\varepsilon)$ are endogenous and determined by the model's parameters, including the firm's optimization problem and the stochastic process for productivity.

Proof. See Appendix A.1. ■

Many models of heterogeneous firms assume a continuum of agents ($M \rightarrow \infty$). In the absence of aggregate shocks, this implies that idiosyncratic shocks average out due to the law of large numbers. However, with a finite number of firms, this is not the case.

Proposition 1. Let aggregate output be defined as $Y_t = \sum_{i=1}^M y_{i,t}$. Then, aggregate output Y_t can be approximated by a log-normal distribution, $\ln(Y_t) \sim \mathcal{N}(\mu_Y, \sigma_Y)$, where:

$$\mu_Y = \ln M + \mu_y + \frac{\sigma_y^2}{2} - \frac{\sigma_Y^2}{2} \quad (6)$$

$$\sigma_Y^2 = \ln \left[\frac{e^{\sigma_y^2} - 1}{M} + 1 \right] \quad (7)$$

with μ_y and σ_y^2 being the parameters from Lemma 1. The coefficient of variation of aggregate output is:

$$\text{CV}(Y) = \frac{\mathbb{V}(Y)^{\frac{1}{2}}}{\mathbb{E}(Y)} = \sqrt{\frac{e^{\sigma_y^2} - 1}{M}}. \quad (8)$$

As with Lemma 1, the distribution of $\ln(k)$ and the covariance between $\ln(k)$ and $\ln(\varepsilon)$ are endogenous and determined by the model's parameters.⁸

Proof. See Appendix A.2. ■

With a finite number of firms ($M < \infty$), the coefficient of variation of aggregate output, given by Equation (8), is strictly positive. This non-zero variation implies that aggregate variables become random, as idiosyncratic shocks do not cancel out in the cross-section. With a finite number of firms, the Law of Large Numbers does not apply, and the period t aggregate output is the sum of the realized output of each firm, not a theoretical mean.

The coefficient of variation, $\text{CV}(Y)$, is influenced by several factors. First, the number of firms (M): $\text{CV}(Y)$ is inversely related to M . As M increases, the coefficient of variation decreases, reflecting the (incomplete) averaging out of idiosyncratic shocks. Second, the variance of idiosyncratic output (σ_y^2): A higher σ_y^2 , reflecting greater heterogeneity in firm-level productivity and capital, positively correlates with $\text{CV}(Y)$. This can be interpreted as reflecting a more skewed firm size distribution, where a few large firms have a disproportionate impact on the aggregate, as originally argued by Gabaix (2011). Finally,

⁸Note that the results are not dependent on the specific investment decision rule. We could, in principle, assume an arbitrary joint log-normal distribution between productivity and capital, and the core results regarding the randomness of aggregate output with a finite number of firms would still hold. However, by starting with a production function and a stochastic process for productivity, we provide a more micro-founded basis for the analysis.

the covariance between capital and productivity (within σ_y^2): As shown in Lemma 1, σ_y^2 incorporates the covariance between capital and productivity. Higher covariance amplifies economic fluctuations and lower covariance dampens economic fluctuations.

The key takeaway is that aggregate output becomes a random variable, even in the absence of aggregate shocks, due to the incomplete averaging out of idiosyncratic shocks. This partial equilibrium analysis highlights the critical role of capital allocation, as reflected in the covariance between productivity and capital, in determining aggregate output volatility.

However, the model takes key prices as given. To move towards a more comprehensive understanding, we now transition to a dynamic general equilibrium framework, which will allow us to explicitly model firms' responses to irreversibilities, endogenize the capital allocation process, and quantify the crucial productivity-capital alignment—as well as its indirect general equilibrium consequences, thereby offering a quantitative assessment of its impact on aggregate fluctuations.

3 General Equilibrium Model

This section develops a quantitative heterogeneous firm business cycle model to assess the impact of idiosyncratic shocks and investment irreversibility on capital misallocation and aggregate fluctuations. We build upon the standard framework where firms use capital and labor to produce a homogeneous good, and where capital is chosen one period in advance. The key departure from the standard model is that we allow the distribution of firm-level productivities to fluctuate stochastically over time, even though the underlying ergodic productivity process for individual firms is a standard Markov chain. This allows us to study the effects of idiosyncratic shocks that do not average out in the aggregate. Importantly, we maintain a rational expectations framework: agents understand the stochastic process governing the evolution of the productivity distribution and form expectations accordingly. This allows us to use recursive methods for the analysis.

3.1 Production and Investment

There is a continuum of firms in the model economy. Each firm undertakes production using a predetermined capital stock k , and labor n . Firms are heterogeneous in their idiosyncratic productivity, ε . The ergodic process for idiosyncratic productivity, ε , is a Markov chain with a finite number of states: $\varepsilon \in \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_{N_\varepsilon-1}, \varepsilon_{N_\varepsilon}\}$. The transition probabilities for this individual firm productivity process are given by $Pr(\varepsilon' = \varepsilon_i | \varepsilon = \varepsilon_j) \equiv \Pi^\varepsilon(\varepsilon_j, \varepsilon_i) \geq 0$, with the standard property that $\sum_{i=1}^{N_\varepsilon} \Pi^\varepsilon(\varepsilon_j, \varepsilon_i) = 1$ for all $j = 1, 2, \dots, N_\varepsilon$.

At the beginning of each period, firms observe the current aggregate state of the economy and choose their current level of employment, n . Production then takes place. After production, firms make their investment decisions, subject to a partial irreversibility friction, represented by the parameter ψ .

The aggregate state of the economy is characterized by the distribution of firms across idiosyncratic productivity and capital, denoted by $\mu(\varepsilon, k)$. This distribution is defined over the space $\mathbf{S} = \mathbb{R}_+ \times \mathbb{R}_+$. Unlike standard models where this distribution is stationary, we allow $\mu(\varepsilon, k)$ to vary stochastically over time. This captures the idea that idiosyncratic shocks do not perfectly cancel out in the cross-section due to the finite number of firms. The evolution of μ is the sole source of aggregate uncertainty in the model. We model this evolution as a Markov process, with $\mu' \sim G(\mu)$, meaning that the distribution in the next period, μ' , is drawn from a distribution G that depends on the current distribution, μ .

Crucially, we define a conditional transition probability for the idiosyncratic productivity process, conditional on both the current and future aggregate state: $Pr(\varepsilon' = \varepsilon_i | \varepsilon = \varepsilon_j, \mu') \equiv \Pi_{\mu'|\mu}^\varepsilon(\varepsilon_j, \varepsilon_i) \geq 0$. This transition probability also satisfies $\sum_{i=1}^{N_\varepsilon} \Pi_{\mu'|\mu}^\varepsilon(\varepsilon_j, \varepsilon_i) = 1$ for all $j = 1, 2, \dots, N_\varepsilon$. The transition probability $\Pi_{\mu'|\mu}^\varepsilon(\varepsilon_j, \varepsilon_i)$ depends on both μ and μ' because, while the underlying ergodic process for ε is exogenous, the realized distribution of firms across productivity levels in the next period depends on the evolution of the aggregate state.⁹

⁹Imagine a simple example: if μ' represents a state with a much higher concentration of firms at high productivity levels than μ , then the probability of a firm transitioning from a low ε to a high ε' will be higher

We can now state the firm's dynamic optimization problem. Let $V(\varepsilon, k; \mu)$ be the value of a firm at the beginning of a period with idiosyncratic productivity ε and predetermined capital stock k , when the aggregate state of the economy is μ . Firms discount future values using a state-contingent discount factor $d(\mu, \mu')$. The firm's problem is:

$$V(\varepsilon, k; \mu) = \max_{n, k'} \varepsilon F(k, n) - \omega(\mu)n - i + \sum_{\mu'} \Pi^{\mu}(\mu'|\mu) d(\mu, \mu') \sum_{\varepsilon'} \Pi_{\mu'|\mu}^{\varepsilon}(\varepsilon'|\varepsilon) V(\varepsilon', k'; \mu') \quad (9)$$

subject to

$$\hat{i} = k' - (1 - \delta)k \quad (10)$$

$$i = \hat{i}(1 - \psi \mathbb{1}_{\hat{i} < 0}) \quad (11)$$

$$k', n \in \mathcal{R}_+ \quad (12)$$

Equation (9) is the Bellman equation. The firm chooses labor, n , and next period's capital stock, k' , to maximize the sum of current profits ($\varepsilon F(k, n) - \omega(\mu)n - i$) and the discounted expected future value. $\omega(\mu)$ is the wage rate, which depends on the aggregate state μ . Equation (10) defines gross investment, \hat{i} , as the difference between next period's capital and depreciated current capital. Equation (11) defines investment, i , incorporating the irreversibility friction: if gross investment is negative (disinvestment), the firm receives only a fraction $(1 - \psi)$ of the value of the capital it sells. Equation (12) imposes non-negativity constraints on capital and labor. The term $\Pi^{\mu}(\mu'|\mu)$ represents the probability of transitioning from the current aggregate state μ to the future aggregate state μ' .

As in Khan and Thomas (2013), the optimal capital decision rule takes an (S, s) form:

under $\Pi_{\mu'|\mu}^{\varepsilon}$ than under the ergodic transition matrix Π^{ε} . The probability $\Pi_{\mu'|\mu}^{\varepsilon}$ captures how the aggregate shock (the change from μ to μ') affects the realized transitions of firms between productivity levels. This is distinct from the time-invariant, ergodic transition probabilities Π^{ε} , which govern the underlying stochastic process for an individual firm's productivity.

$$K(\varepsilon, k; \mu) = \begin{cases} k_u^*(\varepsilon, k; \mu), & \text{if } k \leq \frac{k_u^*(\varepsilon, k; \mu)}{(1-\delta)} \\ k(1-\delta), & \text{if } k \in \left[\frac{k_u^*(\varepsilon, k; \mu)}{(1-\delta)}, \frac{k_d^*(\varepsilon, k; \mu)}{(1-\delta)} \right] \\ k_d^*(\varepsilon, k; \mu), & \text{if } k \geq \frac{k_d^*(\varepsilon, k; \mu)}{(1-\delta)} \end{cases} \quad (13)$$

Here, $K(\varepsilon, k; \mu)$ represents the optimal choice of next period's capital stock, given current productivity ε , current capital k , and the aggregate state μ . The firm has two target capital levels: an upward target, k_u^* , and a downward target, k_d^* . If the firm's current capital is below the inaction region (determined by the targets and the depreciation rate), it invests to reach the upward target. If current capital is above the inaction region, it disinvests (subject to the irreversibility constraint) to reach the downward target. If current capital is within the inaction region, the firm simply lets its capital depreciate.

The target capital levels are defined as:

$$k_u^* = \arg \max_{k'} -k' + \sum_{\mu'} \Pi^\mu(\mu'|\mu) d(\mu, \mu') \sum_{\varepsilon'} \Pi_{\mu'|\mu}^\varepsilon(\varepsilon'|\varepsilon) V(\varepsilon', k'; \mu') \quad (14)$$

$$k_d^* = \arg \max_{k'} -(1-\psi)k' + \sum_{\mu'} \Pi^\mu(\mu'|\mu) d(\mu, \mu') \sum_{\varepsilon'} \Pi_{\mu'|\mu}^\varepsilon(\varepsilon'|\varepsilon) V(\varepsilon', k'; \mu') \quad (15)$$

Equations (14) and (15) show that the target capital levels are chosen to maximize the expected discounted future value of the firm, taking into account the cost of investment (or disinvestment) and the transition probabilities for both the aggregate state and the firm's idiosyncratic productivity.

3.2 Households

Our model economy is populated by a unit measure of identical households. Their lifetime expected utility maximization problem is:

$$W(\lambda; \mu) = \max_{c, n^h, \lambda'} U(c, 1 - n^h) + \beta \mathbb{E}[W(\lambda'; \mu') | \mu] \quad (16)$$

$$\begin{aligned} \text{subject to: } & c + \int_{\mathbf{S}} \rho_1(\varepsilon', k'; \mu) \lambda' (d[\varepsilon \times k']) \\ & \leq w(\mu) n^h + \int_{\mathbf{S}} \rho_0(\varepsilon, k; \mu) \lambda (d[\varepsilon \times k]). \end{aligned} \quad (17)$$

Households hold one-period shares in firms, denoted by λ . Given the prices—the real wage, $w(\mu)$, and the prices of shares, $\rho_0(\varepsilon, k; \mu)$ and $\rho_1(\varepsilon', k'; \mu)$ —households choose their current consumption, c , hours worked, n^h , and the number of new shares, λ' . Let $C(\lambda; \mu)$ and $N(\lambda; \mu)$ represent the household decision rules for consumption and hours worked, respectively, and let $\Lambda(\varepsilon, k', \lambda; \mu)$ be the household decision rule for shares purchased in firms that will begin the next period with k' capital and current productivity ε .

3.3 Recursive Equilibrium

A recursive competitive equilibrium is a set of functions:

$$\begin{aligned} \text{prices} & : (\omega, d, \rho_0, \rho_1) \\ \text{quantities} & : (N, K, C, N^h, \Lambda^h) \\ \text{values} & : (V, W) \end{aligned}$$

that solve firm and household problems and clear the markets for assets, labor, and output as follows:

1. V satisfies equation (9), and (N, K) are the associated policy functions for firms.
2. W satisfies equations (16) and (17), and (C, N^h, Λ^h) are the associated policy functions

for households.

3. $\Lambda^h(\varepsilon, k; \mu) = \mu(\varepsilon, k)$ for each $(\varepsilon, k) \in \mathcal{S}$. (Asset Market Clearing: Households' shareholdings equal the distribution of firms.)
4. The labor and goods markets clear:

$$\begin{aligned} N^h(\mu) &= \int_{\mathbf{S}} N(\varepsilon, k; \mu) \cdot \mu(d[\varepsilon \times k]) \\ C(\mu) &= \int_{\mathbf{S}} [\varepsilon F(k, N(\varepsilon, k; \mu)) - (K(\varepsilon, k; \mu) - (1 - \delta)k)] \cdot Q[K(\varepsilon, k; \mu), k] \cdot \mu(d[\varepsilon \times k]) \end{aligned}$$

where $Q[K(\varepsilon, k; \mu), k]$ is an indicator function that takes value 1 if $K(\varepsilon, k; \mu) > (1 - \delta)k$, and value $(1 - \psi)$ otherwise.

5. The resulting individual decision rules for firms and households are consistent with the conditional distribution of the aggregate state, Π^μ .

3.4 Solution Method

The computational algorithm builds upon the solution method of Khan and Thomas (2008), itself based on Krusell and Smith (1998). It involves approximating the aggregate state space and solving the firm's problem recursively. A key element is the construction of a Markov chain to approximate the stochastic process for the distribution of firms across productivity transitions.

The main challenge to solving traditional heterogeneous agent model is that the aggregate state vector $\mu(\varepsilon, k)$ contains the cross-sectional distribution of firms, which is an infinite-dimensional object. Krusell and Smith (1998) and Khan and Thomas (2008) approximate the intractable cross-sectional distribution μ with a finite dimensional aggregate state variable, such as some moments of the distribution of capital, to make the model solution computable. However, as previously demonstrated in Section 2, the solution of granular economy models must confront an additional key challenge: the cross-sectional distribution μ is not only an infinite-dimensional entity but also inherently stochastic. Indeed, as shown in Section

2, the failure of the law of large numbers induces stochastic fluctuations in the marginal distribution of firms over productivity, $\mu_\varepsilon(\varepsilon)$. For this reason, we must characterize the aggregate endogenous state not only by moments of the capital distribution but also by the distribution of firms across productivity transitions, h , $N_\varepsilon \times N_\varepsilon$ arrays, where each element $h(\varepsilon_i, \varepsilon_j)$ represents the mass of firms that transitioned from productivity state ε_j in the previous period (-1) to productivity state ε_i in the current period.¹⁰ Importantly, distribution of firms across productivity transitions embodies the information of the current and previous period marginal distribution of firm over productivity, as well as the information of the conditional transitional probability for idiosyncratic productivity, $\Pi_h^\varepsilon(\varepsilon_i | \varepsilon_j)$.¹¹ Accordingly, the main problem to solving granular heterogeneous firm model is the construction a feasible set of distribution of firms across productivity transitions, H , and its transition probability, Π^h . To make the problem computationally tractable, we approximate this process with a finite-state Markov chain.¹² In particular, to be consistent with the definition of granularity, we construct the set of possible aggregate productivity distributions, H , and their transition matrix, Π^h , to be consistent with the underlying ergodic productivity process, Π^ε . The key idea is to first approximate the time series of the marginal productivity distribution with a Markov chain by employing the Terry and Knotek II (2011)'s methodology, then use this to construct the transition probabilities for the joint distribution, h . The detailed description of the implementation of our numerical method to build the set of transition probability and its probability space is provided in Appendix C.

¹⁰In Appendix B, we show that we can exactly recover μ with (h, K) in a frictionless economy.

¹¹We can derive the current marginal distribution of productivity, denoted by $\mu_\varepsilon(\varepsilon)$, by summing across the previous period's states: $\mu_\varepsilon(\varepsilon_i) = \sum_j h(\varepsilon_i, \varepsilon_j)$. Similarly, the previous period's marginal distribution is $\mu_{\varepsilon,-1}(\varepsilon_j) = \sum_i h(\varepsilon_i, \varepsilon_j)$. Furthermore, from the distribution of firms across productivity transitions h , we can also derive the conditional transition probability:

$$\Pi_h^\varepsilon(\varepsilon_i | \varepsilon_j) = \frac{h(\varepsilon_i, \varepsilon_j)}{\sum_f h(\varepsilon_f, \varepsilon_j)}, \quad \forall j, i = 1, 2, \dots, N_\varepsilon. \quad (18)$$

¹²Let $H = \{h_1, \dots, h_{N_h}\}$ be a set of N_h possible realizations of the h array. The transitions between these realizations are governed by a transition matrix Π^h , where $\Pi^h(h_l, h_m) = \Pr(h' = h_m | h = h_l) \geq 0$ and $\sum_{m=1}^{N_h} \Pi^h(h_l, h_m) = 1, \forall l = 1, 2, \dots, N_h$.

4 Quantitative Analysis

4.1 Parameters

This section describes our calibration approach. We first specify the functional forms for preferences and technology, then divide parameters into two groups: predetermined parameters set based on standard values in the literature, and calibrated parameters chosen to match key moments from both micro and macro data. We conclude by discussing how well the calibrated model fits both targeted and untargeted moments.

4.1.1 Model Specification

We assume that the representative household's period utility follows the indivisible labor specification of Hansen (1985) and Rogerson (1988): $u(c, n^h) = \ln(c) + \eta(1 - n^h)$. The firm-level production function takes a Cobb-Douglas form with decreasing returns to scale: $\varepsilon F(k, n) = \varepsilon k^\alpha n^\nu$, where $\alpha + \nu < 1$. Productivity is the product of two components: $\varepsilon = \xi \cdot \epsilon$, where ξ is a persistent base component and ϵ is a transitory component. In each period, the base component ξ is redrawn with probability χ from a Pareto distribution $\mathcal{P}(e_m, e)$.¹³ With probability $(1 - \chi)$, the base component remains unchanged. The transitory part follows a log-normal process: $\ln(\epsilon) \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

The assumption that ξ is redrawn from a Pareto distribution with probability χ serves a dual purpose in our model. First, it generates the fat-tailed firm size distribution observed empirically, similar to Ehouarne et al. (2022).¹⁴ Second, it captures the pervasive micro-level rank reversals among firms as even major corporations can experience dramatic sales declines, “micro disasters,” that fundamentally alter their market position (Autor, Dorn,

¹³The Pareto distribution $\mathcal{P}(e_m, e)$ has probability density function $f(\xi) = \frac{e \cdot e_m^e}{\xi^{e+1}}$ for $\xi \geq e_m$, where $e_m > 0$ is the scale parameter (minimum value) and $e > 0$ is the shape parameter determining the tail thickness.

¹⁴Ehouarne et al. (2022) shows that permanent idiosyncratic shocks combined with a power law distribution in firm size are crucial for generating time-variation in common idiosyncratic skewness (see also Salgado et al. (2019)).

Katz, Patterson, and Van Reenen, 2020). This modelling approach is also consistent with recent evidence by Melcangi and Sarpietro (2025), who document that firm-level sales and productivity are hit by heavy-tailed shocks.

4.1.2 Calibration Strategy

We set the length of a period in the model to be one year. The parameters fall into two categories:

Predetermined parameters: We set five parameters based on standard values in the literature. The household discount factor, $\beta = 0.96$, implies an average real interest rate of four percent (Gomme, Ravikumar, and Rupert, 2011). The labor share, $\nu = 0.60$, matches the average labor share of income (Cooley, 1995). The depreciation rate of capital, $\delta = 0.07$, targets the average investment-to-capital ratio. The standard deviation of the transitory component, $\sigma_\epsilon = 0.022$, follows (Khan and Thomas, 2008). Finally, we set the number of firms $M = 4.5$ million to match the approximate number of firms in the US economy.

Calibrated parameters: The remaining six parameters are jointly calibrated to match key moments from the data. These parameters are: (1) the capital share, α , (2) the preference parameter, η , (3) the shape parameter of the Pareto distribution, e , (4) the scale parameter of the Pareto distribution, e_m , (5) the probability of resetting the base component, χ , and (6) the degree of investment irreversibility ψ .

We target the following six data moments: (1) the aggregate capital-output ratio of 2.3 as the average private capital-to-output ratio between 1954 and 2002 reported by Khan and Thomas (2013), (2) aggregate total hours worked of one-third, (3) standard deviation of firm-level TFP growth among the 100 largest firms of 12% (Gabaix, 2011), (4) the sales share of largest 500 firms of 40% (Gabaix, 2011), (5) the largest firm's employment share of 1% (Walmart's domestic employment size of 1.4 million as in (Carvalho and Grassi, 2019)), and (6) inaction rate defined as the mass of firms among the 100 largest firms of the previous

period whose $|i/k| \leq 0.01$.¹⁵

4.1.3 Results

Tables 1 and 2 present our calibration results. Table 1 shows both targeted and untargeted moments, while Table 2 reports the calibrated parameter values.

Table 1: Moments

Targeted Moments				
Description	Parameter Sensitivity	Data	Model	
Capital-Output Ratio ($\frac{K}{Y}$)	α, e_m, χ	2.300	2.23	
Total Hours Worked (N)	η	0.333	0.336	
TOP 100 Std. Dev. $\Delta\varepsilon$ (perc.)	e, e_m	12.00	11.90	
Sales share TOP 500 firms (perc.)	e_m	40.00	39.50	
Walmart Employment Share (perc.)	e, e_m	1.000	1.087	
$Pr(\frac{i}{k} \leq 0.01)$	ψ, α, e	0.08 – 0.111	0.097	
Untargeted Moments				
Tail index of Firm size dist.		1.097	1.078	
Sales share TOP 50 Firms (perc.)		24.00	22.40	
Sales share TOP 100 Firms (perc.)		29.00	27.55	
$Pr(\frac{\Delta k'}{k} \leq -0.2)$ previous TOP 500 firms		0.024	0.022	

Note: The “Parameter Sensitivity” column indicates which parameters primarily affect each targeted moment, not a one-to-one mapping.

The calibration yields several important findings. First, the calibrated degree of capital irreversibility ($\psi = 0.35$) aligns closely with the empirical findings of Bloom (2009).

¹⁵We target an intermediate value between 0.080 of Khan and Thomas (2008) and 0.111 obtained from Compustat. The investment rate is given by the ratio of capital expenditures (CAPX) to lagged PPEGT after controlling for SECTOR \times YEAR. See Appendix D for further details on variable construction.

Second, the reset probability $\chi = 0.02$ generates realistic firm dynamics. As Autor et al. (2020) document, the probability that a firm remains in the top 500 after one year is 0.95, which our model matches precisely with $1 - \chi = 0.98$ for annual transitions. Third, the model successfully replicates the firm size distribution. The estimated tail index (1.078) closely matches the empirical value of 1.097 reported by Carvalho and Grassi (2019). Furthermore, the model accurately reproduces the right tail of the distribution: the output shares of the top 50 and top 100 firms are very close to the empirical estimates provided by Gabaix (2011). The model’s ability to replicate the skewness of the firm size distribution is particularly important, as this feature is essential for generating granular volatility—an insight originally emphasized by Gabaix (2011). The strong performance on untargeted moments provides validation of our modelling approach. These moments—particularly the tail index and sales concentration measures—were not directly targeted in our calibration but emerge naturally from the model’s structure. This suggests that our productivity process with Pareto-distributed resets captures fundamental features of firm dynamics. Finally, the model replicates the negative net investment spikes observed among the largest firms in the economy, indicating that the calibrated stochastic process generates realistic micro-level investment dynamics.

Table 2: Parameter values

Pre-determined parameters		
Description	Parameter	Value
Discount factor	β	0.96
Labor share	ν	0.60
Depreciation rate	δ	0.07
Std. dev. transitory component	σ_ϵ	0.022
Number of firms	M	4.5 mil.

Calibrated parameters		
Capital share	α	0.26
Preference parameter	η	2.15
Probability of reset	χ	0.02
Pareto scale parameter	e_m	6.89
Curvature of Pareto distribution	e	6.90
Investment irreversibility	ψ	0.35

4.2 Business Cycles

We now examine how capital irreversibility shapes key business cycle dynamics in our granular economy, distinguishing between its direct impacts on firm behavior and its broader general equilibrium consequences.

Table 3: Business Cycle Moments Benchmark Model

	$\psi = 0.35$			$\psi = 0.00$			DATA		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, \ln Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, \ln Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$\rho(x, \ln Y)$
$\ln Y$	0.294	1.000	1.000	0.377	1.000	1.000	1.299	1.000	1.000
$\ln C$	0.171	0.582	0.786	0.119	0.316	0.716	1.039	0.800	0.899
$\ln I$	1.200	4.088	0.901	1.962	5.209	0.975	5.913	4.552	0.916
$\ln H$	0.191	0.651	0.834	0.303	0.805	0.962	1.700	1.309	0.881
$\ln(1+r)$	0.025	0.086	-0.496	0.018	0.049	-0.022	1.310	1.007	-0.286

Notes: The table shows the data, benchmark ($\psi = 0.35$), and frictionless ($\psi = 0.00$) model equilibrium business cycle moments of output Y , consumption C , investment I , and hours worked H . $\sigma(x)$ is the standard deviation of x , $\sigma(x)/\sigma(Y)$ is the relative standard deviation to that of Y , and $\rho(x, \ln Y)$ is the contemporaneous correlation of x with Y . The model moments are obtained from a 15,000-period unconditional simulation using the solution of the model. All series are HP-filtered in logs with a smoothing parameter of 6.25, following Ravn and Uhlig (2002). Data sources for the reported moments are: (1) real gross domestic product (GDPCA taken from FRED), (2) real gross private domestic investment (GPDICA taken from FRED), (3) real personal consumption expenditures (PCECCA taken from FRED), (4) total nonfarm business sector hours (HOANBS taken from FRED but annualized), and (5) nominal return on 1-year Treasury bills (DGS1 taken from FRED) adjusted for realized CPI inflation (CPIAUCSL_PC1 taken from FRED). The sample refers to the period spanning from 1964 to 2019.

Before examining the specific channels through which capital irreversibility operates, it is worth noting that our granular economy generates business cycle dynamics similar to those of a typical real business cycle model with aggregate TFP shocks. As shown in Table 3, the model captures key empirical regularities: investment is substantially more volatile than output ($\sigma(\ln I)/\sigma(\ln Y) = 4.088$), consumption is less volatile than output (0.582 versus 0.800), and there are strong positive contemporaneous correlations with output in consumption, investment, and total hours worked. These patterns emerge from granular shocks—idiosyncratic productivity fluctuations at large firms—rather than aggregate TFP shocks, consistent with the seminal findings of Carvalho and Grassi (2019), who show that granular shocks can generate realistic business cycle dynamics.¹⁶

¹⁶This similarity is noteworthy given several key differences in the modeling frameworks. For instance, a crucial distinction is our model’s general equilibrium nature, where the wage endogenously adjusts to clear the labor market. This GE channel, which we analyze in detail in Section 4.2.2, acts as a stabilizer affecting aggregate volatility. That both models generate similar business cycle dynamics, despite these

Another thing to note is that the model generates real interest rate dynamics different from a typical real business cycle model with aggregate TFP shocks. Also, the correlation of the real interest rates with output depends on the degree of capital irreversibility.¹⁷ Without irreversibility ($\psi = 0.00$), the real interest rate is essentially acyclical ($\rho(\ln(1+r), \ln Y) = -0.022$), while with irreversibility ($\psi = 0.35$), it becomes moderately counter-cyclical ($\rho(\ln(1+r), \ln Y) = -0.496$). This sensitivity to capital market frictions reflects how irreversibility constrains both firms' capital adjustments and households' consumption smoothing, affecting equilibrium interest rate determination through precautionary savings motives.¹⁸

4.2.1 Direct Effects on Investment and Capital Allocation

To understand this result, we first decompose the mechanisms at play, beginning with the direct effects of irreversibility on firm-level behavior. Capital irreversibility (ψ) plays a significant role in shaping aggregate volatility in our granular economy, primarily through its *direct effects* on firm-level investment and capital allocation. As evidenced in Table 3, overall output volatility, $\sigma(\ln Y)$, decreases as the degree of capital irreversibility (ψ) increases, falling from 0.377 in the frictionless model ($\psi = 0.00$) to 0.294 in the benchmark model with irreversibility ($\psi = 0.35$).¹⁹ This can be understood through key *direct mechanisms*. Firstly,

foundational differences, underscores the robustness of the granular shock mechanism. To illustrate the quantitative importance of this GE wage channel, Table 7 in Appendix E presents business cycle moments for our benchmark, frictionless, and partial equilibrium specifications side-by-side.

¹⁷Given the aggregate state μ , the real interest rate is computed as the expected consumption growth: $r = \frac{\sum_{\mu'} \Pi^\mu(\mu'|\mu) C'(\mu')}{C(\mu)} - 1$. As shown in Appendix C.2, by approximating the endogenous aggregate state with moments of the endogenous marginal distribution of k and h , we derive C' using a log-linear forecasting rule.

¹⁸Appendix F, Table 8, shows that the real interest rate is countercyclical when considering the 1964–2019 sample. Although it appears procyclical in the 1980–2019 sample, the degree of procyclicality remains substantially lower than that predicted by standard real business cycle models, wherein persistent improvements in technology typically induce simultaneous increases in consumption and investment demand, which, in the short run, outpace supply and generate procyclical movements in the real interest rate (Beaudry and Guay, 1996). See also Winberry (2021) for the recent contribution on the cyclicalities of the real interest rates in a real business cycle model.

¹⁹Eliminating partial investment irreversibility can have two countervailing effects on the shape coefficient of the firm size distribution. On the one hand, irreversibility limits firms' ability to shrink, creating persistence and path dependence in firm size; removing this friction allows less productive firms to contract more readily, thinning the upper tail and increasing the shape coefficient. On the other hand, greater responsive-

higher irreversibility *directly* causes firm-level capital stocks to adjust more sluggishly to idiosyncratic productivity shocks. This reduced responsiveness is a key channel, leading to a lower covariance between capital and current productivity. As stated in Proposition 1, in a granular economy, such a dampened alignment of capital with productivity contributes to reducing aggregate output volatility. This dampened alignment is evident in our simulation results: the average log dispersion of the marginal product of capital across large firms ($\ln \sigma^{MPK,200}$) increases from 0.019 in the frictionless model to 0.022 in our benchmark model with irreversibility. Investment fluctuations are also dampened under irreversibility. Table 3 shows that aggregate investment volatility, $\sigma(\ln I)$, drops from 1.962 when $\psi = 0.00$ to 1.210 when $\psi = 0.35$, making it far less responsive to shocks. This finding, that irreversibilities dampen changes in aggregate investment, is similar to the work of Lanteri (2018), Blanco and Baley (2024) and Khan and Thomas (2013), while it is different from the irrelevance result from standard real business cycle models driven solely by aggregate TFP shocks. In such settings, capital irreversibility often has a much more limited effect on aggregate output volatility. A set of business cycle statistics for a comparable economy driven by aggregate TFP shocks are in Appendix H, with key moments in Table 10 therein.

4.2.2 Indirect General Equilibrium Effects: The Wage Channel

Beyond these direct impacts on firm behavior and capital allocation, irreversibility also triggers *indirect general equilibrium effects* that further shape aggregate dynamics, particularly through wage adjustments. When capital adjustment is restricted, the representative household's ability to smooth consumption over time is diminished. Indeed, Table 3 shows that consumption volatility, $\sigma(\ln C)$, increases from 0.119 in the frictionless model ($\psi = 0.00$) to 0.170 with irreversibility ($\psi = 0.35$). This suggests more pronounced movements in wages,

ness to shocks in the frictionless economy may increase firm-level volatility, which tends to fatten the tail and reduce the shape coefficient. In our model, the first effect dominates: the shape coefficient in the frictionless economy is 1.139, indicating a thinner-tailed, more equal firm size distribution.

as seen in the intra-temporal optimal condition for the household's labor decision:

$$\omega(\mu) = \eta C(\mu), \quad (19)$$

where ω represents the real wage, C is aggregate consumption, and η is a preference parameter. When households cannot effectively smooth consumption through capital adjustments, they become more responsive to shocks in their labor supply decisions. Following negative shocks, the sharper decline in aggregate consumption (C) directly translates into a more substantial decrease in the real wage (ω) through the above equation. Households facing this consumption drop are more willing to accept lower wages to maintain employment, creating more elastic labor supply. This greater wage flexibility, in turn, tempers the reduction in aggregate output. Firms facing negative shocks benefit from a larger reduction in wage costs, which can mitigate the decline in employment and production. This is consistent with the observed decrease in the volatility of hours worked, $\sigma(\ln H)$, from 0.303 to 0.193 (Table 3), and contributes to the overall reduction in output volatility to 0.294 in the benchmark model.

The overall reduction in output volatility due to capital irreversibility in our granular economy stems from both the direct channel (detailed in Section 4.2.1) and the indirect general equilibrium channel detailed here. A quantitative decomposition of the relative contributions of these direct and indirect effects (GE wage) is provided in Appendix G (see Table 9 there).

4.3 Dynamics of Micro-level Investment Moments

The preceding analysis established that capital irreversibility dampens aggregate output volatility in our granular economy. To understand more deeply why and how irreversibility exerts such a significant influence when interacted with granular shocks—a contrast to its often muted role in canonical real business cycle models driven solely by aggregate TFP shocks—we now examine the micro-level investment behavior. Table 4 presents key statistics

on firms' investment dynamics, comparing our benchmark granular economy with a canonical real business cycle economy featuring only aggregate TFP shocks.

Table 4: Micro Investment Behavior (Values for x and $\sigma(x)$ in percent)

Micro-moment	Granular Economy			RBC Economy		
	x	$\sigma(x)$	$\rho(x, \ln Y)$	x	$\sigma(x)$	$\rho(x, \ln Y)$
$Pr(\frac{i}{k} \leq 0.01)$	9.7	0.2	0.060	8.9	0.2	0.318
$Pr(\frac{i}{k} \leq 0.01)$ TOP 500	5.4	1.2	-0.260	5.4	0.2	-0.018
$Pr(\frac{\Delta k'}{k} \leq -0.20)$	0.4	0.0	0.184	0.4	0.0	-0.295
$Pr(\frac{\Delta k'}{k} \leq -0.20)$ TOP 500	2.2	0.4	-0.525	2.2	0.0	-0.518
$Pr(i \leq 0.000)$	0.7	0.0	0.049	1.6	0.0	-0.696
$Pr(i \leq 0.000)$ TOP 500	3.5	1.7	-0.123	3.3	0.6	0.021
$Pr((1 - \delta)k \geq k_u^*)$	10.3	0.0	0.106	10.3	0.0	-0.183
$Pr((1 - \delta)k \geq k_u^*)$ TOP 500	7.7	0.8	-0.357	7.7	0.0	-0.063

Notes: The table compares the ergodic value and cyclical of micro investment behavior in the benchmark model (Granular Economy) with a traditional model economy whose volatility is driven by aggregate TFP shocks. For each specified micro-moment (investment behavior): The column headed ' x ' reports the mean probability, expressed in percent. The column headed ' $\sigma(x)$ ' reports the standard deviation of this probability, expressed in percentage points. The column headed ' $\rho(x, \ln Y)$ ' reports the contemporaneous correlation of the micro-moment with the logarithm of aggregate output Y . Investment behaviors considered are: inaction rate ($Pr(|\frac{i}{k}| \leq 0.01)$), negative net investment spike rate ($Pr(\frac{\Delta k'}{k} \leq -0.20)$), negative investment rate ($Pr(i \leq 0.000)$), and the probability of the irreversibility constraint binding ($Pr((1 - \delta)k \geq k_u^*)$). These are calculated for both the entire economy and for the largest 500 firms from the previous period.

A critical distinction emerges from Table 4 when observing the cyclical behavior of the irreversibility constraint itself, especially for large firms. In the Granular Economy, the share of the TOP 500 firms that are constrained by irreversibility ($Pr((1 - \delta)k \geq k_u^*)$ – meaning they wish to downsize their capital stock but cannot due to the constraint – exhibits notable volatility ($\sigma(x) = 0.8$ percentage points) and is strongly counter-cyclical ($\rho(x, \ln Y) = -0.357$). This indicates that during downturns, which can be initiated or

amplified by adverse granular shocks hitting these large firms, a significant and cyclically varying number of these economically important firms are actively impeded by irreversibility.

In stark contrast, the Real Business Cycle Economy driven by aggregate TFP shocks shows a different picture. For the TOP 500 firms, the share constrained by irreversibility has virtually no volatility ($\sigma(x) = 0.0$) and a much weaker correlation with the aggregate cycle. While some large firms might be constrained on average, this fraction does not fluctuate meaningfully over the business cycle in response to aggregate shocks alone. This lack of cyclical variation in the mass of constrained large firms is a key reason why irreversibility often appears less consequential in such models.

This difference in the cyclical engagement of the irreversibility constraint for large firms is fundamental. When substantial granular shocks occur, capital irreversibility actively shapes the investment response of a cyclically fluctuating segment of the economy's largest firms. Their inability to disinvest optimally during periods of idiosyncratic distress (or when hit by a large negative component of a granular shock) means that capital may remain temporarily misallocated. However, it also implies that aggregate investment does not contract as sharply as it might otherwise, and productive resources are not shed as precipitously. This micro-level dynamic, particularly the time-varying incidence of binding irreversibility constraints among large firms, allows irreversibility to significantly alter aggregate business cycle dynamics. Canonical real business cycle models with only aggregate shocks typically do not generate sufficient firm-specific shock variance to cause such cyclically sensitive binding of irreversibility for a large, influential group of firms. Granular shocks, by their very nature, provide these large, idiosyncratic disturbances that make the irreversibility constraint a more active margin for key economic players.²⁰

²⁰Further evidence from Table 4 supports this. For instance, the inaction rate ($Pr(|\frac{i}{k}| \leq 0.01)$) among the TOP 500 firms is more volatile ($\sigma(x) = 1.2$ percentage points vs. 0.2 percentage points) and more strongly counter-cyclical ($\rho(x, \ln Y) = -0.260$ vs. -0.018) in the granular economy. Similarly, large disinvestments ($Pr(\frac{\Delta k'}{k} \leq -0.20)$) by TOP 500 firms are also more volatile and counter-cyclical in the presence of granular shocks.

5 Granular Shocks and Capital Misallocation in the data and model

Our theoretical framework argues for a significant role for granular shocks in driving capital misallocation, particularly when investment irreversibility is present. These dynamics, in turn, are argued to be important contributors to aggregate business cycle fluctuations. This section now presents empirical evidence from U.S. firm-level data to substantiate this narrative. We investigate the empirical linkage between identified granular shocks and ensuing changes in capital misallocation, and assess whether these observed dynamics align with the predictions and mechanisms of our structural model.

We first construct an annual panel dataset by merging the firm-level TFP estimates from İmrohoroglu and Tüzel (2014) with Compustat databases from 1964 to 2019.²¹ Similar to Gabaix (2011), we construct the series of granular residual as the sales-weighted idiosyncratic shocks among the 200 largest firms:²²

$$\Theta_t^{Gabaix} = \sum_{i=1}^{100} \frac{\text{Sale}_{i,t-1}}{\text{Sale}_{t-1}^{100}} \Delta \varepsilon_{i,t} - \frac{\sum_{i=1}^{100} \Delta \varepsilon_{i,t}}{100} \quad (20)$$

where the proxy for idiosyncratic productivity shocks is the productivity growth rate $\Delta \varepsilon_{i,t} = \frac{\varepsilon_{i,t} - \varepsilon_{i,t-1}}{\varepsilon_{i,t-1}}$. To build a measure of capital misallocation, we follow Kilic and Tuzel (2025) and we approximate the marginal product of capital by the logarithm of the ratio of sales (SALE) to physical capital (PPEGT).²³ Accordingly, we measure capital misallocation using the dispersion of the marginal product of capital among the largest 200 firms, $\sigma^{MPK,200}$:

$$\sigma_t^{MPK,200} = \sqrt{\frac{\sum_{i=1}^{200} (MPK_{i,t} - MPK_t^{mean})^2}{200}} \quad (21)$$

Finally, we test whether the model can qualitatively and quantitatively replicate how gran-

²¹See Appendix D for the details of the variable constructions.

²²Note that we follow Gabaix and Koijen (2024) and we subtract the equal-weighted shocks from the weighted shocks to ensure the elimination of any common observed and unobserved aggregate factors.

²³Idiosyncratic shocks and marginal product of capital are cleaned for 2-digit sector times year fixed effects.

ular misallocation responds to granular shocks. To this end, we employ Jordà (2005)'s local projection method to estimate impulse responses of granular misallocation to granular shocks. This method is based on the estimation of a series of regressions for each horizon τ for each variable. Accordingly, the linear model is as follows:

$$\ln(\sigma_{t+\tau}^{MPK,200}) = \alpha_\tau + \phi_\tau(L)z_{t-1} + \beta_\tau\Theta_t^{Gabaix} + v_{t+\tau} \quad \text{for } \tau = 0, 1, 2, \dots, \quad (22)$$

where z is a vector of control variables, $\phi_\tau(L)$ is a polynomial in the lag operator, and the shock is the identified granular shock. The shock is the granular residual. Our vector of baseline control variables, z , contains the logarithm of real GDP, 10-year T-bill, term spread (the government 5-year bond yield minus the 1-year yield), consumer price inflation, and the logarithm of the dispersion of the marginal product of capital computed excluding the largest 200.²⁴ Furthermore, z includes lags of the granular shock variable to control for any serial correlation in the shock variable. The term $\phi_\tau(L)$ is a first-order polynomial.

Figure 1 compares the impulse responses of the level and the first difference in the logarithm of the misallocation of granular capital to a negative granular residual of one standard deviation between the model and the data. The model performs well in quantitatively replicating the response of capital misallocation among large firms to granular shocks, as the model IRFs lie within the 95 percent confidence interval of the empirical IRFs. A negative granular shock is associated with negative productivity shocks to larger firms, increasing capital misallocation. Importantly, the discrepancy between the IRFs generated by the model and those observed in the data arises from the model's exclusion of additional frictions that are present in the real economy. These omitted frictions contribute to a higher ergodic dispersion of the marginal product of capital, while simultaneously dampening its responsiveness to granular shocks. As shown in Appendix I, this result is robust across alternative

²⁴In particular, we use the annual real GDP (GDP from FRED). We use yield treasury securities at 10, 5, and 1-year constant maturity (DGS10, DGS5 and DGS1 from FRED). We use consumer price inflation (FPCPITOTLZGUSA from FRED). The dispersion of marginal product of capital excluding the largest 100 firms is obtained from the COMPUSTAT dataset. All series are HP-filtered using a smoothing parameter of 6.25, following Ravn and Uhlig (2002).

sample periods and industry-level controls used in the construction of firm-level MPK.

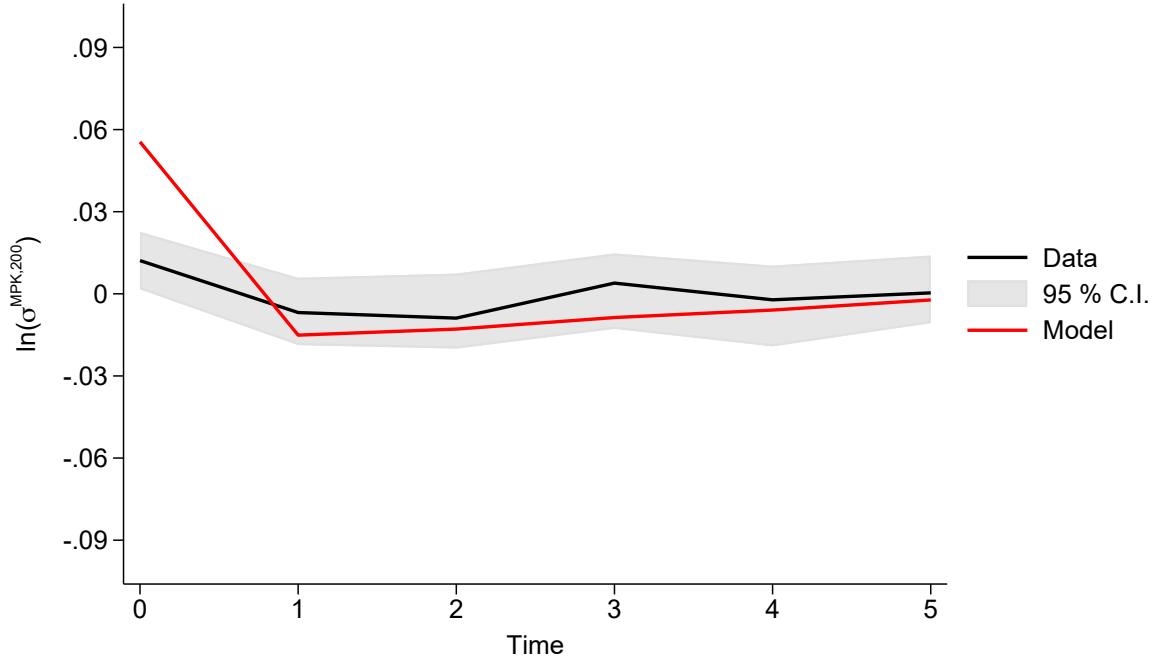


Figure 1: Effect of granular residual shock on granular misallocation.

Notes: Responses of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular residual shock. The black line represents the response estimated from the data. The red line represents the response obtained from the simulated data. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 1000 repetitions. All series are HP-filtered using a smoothing parameter of 6.25, following Ravn and Uhlig (2002). Granular misallocation is estimated conditional on 2-digit sector-by-year fixed effects. For further details on data construction, see Appendix D.

6 Policy Implications

Our preceding analysis has highlighted that investment irreversibility influences business cycle dynamics in a granular economy. While investment irreversibility may dampen fluctuations in aggregate output and investment, leading to seemingly desirable reduced volatility in those aggregate measures, these benefits are counteracted by the costs arising from sub-optimal capital allocation and a deterioration in households' ability to smooth consumption, leading to greater consumption volatility. The overall welfare outcome then becomes ambiguous, necessitating careful policy evaluation that weighs these contrasting effects against

each other.

This issue carries significant policy relevance. Historical interventions, such as the Troubled Asset Relief Program (TARP) in the United States, exemplify attempts to facilitate capital reallocation, particularly from distressed firms.²⁵ As our model demonstrates, policies that foster a better alignment between productivity and the capital stock across firms can inadvertently increase aggregate volatility. Therefore, whether such interventions are welfare-improving remains a critical question. To explore these welfare implications, we evaluate two distinct policy options designed to promote capital reallocation by mitigating the costs associated with investment irreversibility.

Our approach to calculating welfare follows the methodology established by Lucas (1987) and further developed by Krusell and Smith (1999). This involves converting differences in expected lifetime utility into “consumption equivalents” (CEV), representing the permanent percentage increase in consumption that would make a representative household indifferent between the pre-policy and post-policy economies. It is important to note that, unlike the existing studies, our exercise does not aim to eliminate business cycles entirely. Instead, we assess the welfare impact of specific policy interventions within an ongoing granular economy.

6.1 Policy Options

Government Subsidy: In this scenario, we consider a policy where the government fully absorbs the price gap between the cost of purchasing capital and the value recovered from selling it. This subsidy is financed through lump-sum taxation on the representative household. This policy aims to reduce the financial friction associated with asset liquidation, encouraging firms to adjust their capital stock more flexibly.

Eliminate Price Gap: This policy represents a more direct intervention, where the price gap is entirely removed, allowing firms to recover the full value of their capital upon sale. This effectively simulates an environment where capital is perfectly reversible, removing a

²⁵See U.S. Congressional Budget Office (2009) and Rattner (2010) for further details.

pure friction from the economy.

The second policy option—eliminating the price gap entirely—may appear extreme in our stylized model, but it closely reflects actual policy objectives during financial crises. The implementation of TARP, for instance, demonstrated how governments can effectively reduce capital market frictions. By purchasing distressed assets at above-market prices, TARP eliminated the gap between book values and severely depressed market prices, creating a buyer of last resort that restored liquidity to frozen markets. The Treasury’s stated goal was precisely to “restore liquidity and stability to the financial system” by ensuring that “the prices of assets would better reflect their underlying value.”²⁶ This real-world precedent suggests that policies approximating perfect reversibility can be both feasible and necessary during periods of severe market dysfunction, particularly when large, systemically important firms face binding irreversibility constraints.

6.2 Welfare Implications

The long-run welfare gains from these policies, measured in consumption equivalent variation (CEV), are summarized in Table 5.

Table 5: Welfare Implication of Capital Irreversibility

	$E(U)$	CEV (perc.)
<i>Benchmark</i>	24.746	
<i>Subsidy</i>	24.831	0.341
<i>No-Price Gap</i>	25.368	2.520

Under the Government Subsidy policy, the policy intervention results in a consumption-equivalent variation of approximately 0.34%. This implies that the representative household would be indifferent between residing in the initial economy with a permanently higher

²⁶See U.S. Department of the Treasury, Office of Financial Stability (2010).

consumption level of 0.34% and living in the economy with the subsidy policy implemented. As shown in Table 11 of Appendix J, this policy enhances welfare primarily by reducing consumption volatility and, more significantly, by increasing average consumption. While it substantially raises investment volatility, thereby intensifying capital reallocation, its impact on consumption volatility remains limited. This outcome can be attributed to the fact that firms do not internalise the social costs associated with capital reallocation, which may lead to excessive capital destruction through divestment and potentially amplify volatility due to over-reallocation.

Under the Eliminate Price Gap policy, where capital is perfectly reversible (i.e., irreversibility costs are zero), the welfare gain is substantially larger, reaching approximately 2.52% in consumption equivalent variation. In this scenario, the policy is unequivocally efficiency-enhancing, as it removes a pure friction without introducing behavioural distortions. This policy also yields a dramatic reduction in consumption volatility, contributing significantly to the overall welfare improvement.

Our analysis reveals a fundamental insight for policymakers: in granular economies, the welfare implications of investment irreversibility are inherently ambiguous. While irreversibility dampens aggregate volatility—an apparent benefit—it simultaneously impairs allocative efficiency and consumption smoothing. The stark contrast in welfare gains between our two policy scenarios (0.34% versus 2.52%) underscores that the design of interventions matters crucially. Policies that merely subsidize the symptoms of irreversibility without addressing its root causes may inadvertently encourage excessive capital destruction, as firms fail to internalize the social costs of reallocation. In contrast, policies that enhance the fundamental liquidity and efficiency of capital markets can yield substantial welfare improvements. These findings suggest that in economies where large firms drive aggregate fluctuations, policymakers should prioritize reducing frictions in capital reallocation markets—through improved bankruptcy procedures, standardization of assets, or development of secondary markets—rather than simply compensating firms for liquidation losses. The

granular nature of modern economies thus calls for a reconsideration of traditional stabilization policies, with greater emphasis on maintaining efficient capital allocation even during periods of significant idiosyncratic disturbances.

7 Conclusion

When aggregate fluctuations originate from firm-level shocks, irreversibility dampens the response of large firms to negative shocks, reducing the covariance between productivity and capital and, in turn, lowering output and investment volatility. These effects are amplified in general equilibrium: limited reallocation impairs households' ability to smooth consumption, increasing real wage flexibility and further absorbing shocks. Our analysis suggests that while investment irreversibility creates ambiguous welfare effects—dampening volatility but impairing allocative efficiency—the design of interventions matters crucially. Policymakers should prioritize enhancing the fundamental liquidity and efficiency of capital reallocation markets, enhancing allocative efficiency without encouraging excessive capital destruction.

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