A NEW LOOK AT UNCERTAINTY SHOCKS: IMPERFECT INFORMATION AND MISALLOCATION*

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Abstract

Uncertainty faced by individual firms appears to be heterogeneous. In this paper, I construct a new set of empirical measures of firm-level uncertainty using data such as the IBES and Compustat. The panel data that I construct reveals persistent differences in the degree of uncertainty facing individual firms not reflected by existing measures. Consistent with existing measures, I find that the average level of uncertainty across firms is countercyclical, and that it rose sharply at the start of the Great Recession and the Covid-19 pandemic. I next develop a heterogeneous firm model in a setting wherein each firm gradually learns about its own productivity, and each occasionally experiences a shock forcing it to start learning afresh. Uncertainty is gradually resolved as firms operate longer and get better informed. I show that an uncertainty shock can explain a sizable portion of the observed volatility of macroeconomics variables at business cycle frequency.

JEL: E22, E32, D8, D92
Keywords: Uncertainty, learning, misallocation and business cycles.

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1 Introduction

"*Subjective uncertainty is about the "unknown unknowns". When, as today, the unknown unknowns dominate, and the economic environment is so complex as to appear nearly incomprehensible, the result is extreme prudence, [. . .], on the part of investors, consumers and firms.*" Olivier Blanchard (2009)

How large is the role of increased uncertainty in driving economic downturns? Is there a link between a rise in firm-level uncertainty and the subsequent pace of economic recovery? To explore these questions, I construct new empirical measures of firm-level uncertainty, and I show that the degree of uncertainty varies across firms and the average level of uncertainty, as well as its dispersion, across firms is countercyclical. To account for these regularities, I develop a heterogeneous firm model that incorporates learning at the firm level with uncertainty shocks. The model can match both the cross-sectional and time series properties of firm-level uncertainty. In addition, the model successfully reproduces a gradual recovery of the aggregate economy following uncertainty shocks.

A defining feature of this paper is that the uncertainty faced by firms not only varies over time but also varies across firms. One common approach in the uncertainty shock literature, following the seminal work of Bloom (2009), has been to study stochastic volatility models. I break with this tradition primarily because my learning model is well-suited to capture the heterogeneous uncertainty evident in microdata.[1] I integrate Jovanovic (1982)-style of Bayesian learning into an otherwise standard heterogeneous firm business cycle framework. In this model, by contrast, uncertainty, defined as the conditional variance of forecasts of firm performance, varies across firms depending on the information each firm possesses. Firms are heterogeneous in both productivity and their confidence about that productivity; better informed firms have lower posterior variances of their beliefs. Two different firms can have the same posterior mean while differing in their posterior variances. Hence, uncertainty differs across firms. A second appealing feature of the model is the fact that the recession in response to an uncertainty shock is not followed by a sharp recovery, as happens in existing stochastic-volatility-based uncertainty shock models.[2] Instead, my model with a non-trivial distribution of firms with learning drives a slow economic recovery as firms gradually regain information and confidence. Moreover, these results require no additional rigidity or frictions. In the absence of labor and capital adjustment costs, uncertainty shocks still cause recessions.[3]

[1] In a common stochastic volatility approach as in Vavra (2014), Bloom et al. (2018), and Bachmann and Bayer (2013), there is full information and all agents know the true distribution of shocks that they face, including its volatility, which varies over time. In uncertain times, the volatility that every agent faces rises equally.

[2] See, for the discussion, Bachmann, Elstner and Sims (2013) and Bachmann and Bayer (2013).

[3] The large body of literature about the relation between uncertainty and investment studies the real options effect in models with adjustment costs, as in Bertola and Caballero (1994), Dixit and Pindyck (1994), Abel and Eberly (1996) and Caballero and Engel (1999).

I construct a new panel dataset of firm-level uncertainty based on data from the Institutional Brokers' Estimate System (I/B/E/S), Center for Research in Securities Prices (CRSP), OptionMetrics, and Compustat databases. By merging these data, I construct an annual panel of US public firms with uncertainty measures such as an ex-ante earnings forecast dispersion among market analysts, ex-post-realized forecast errors and stock price volatility measures. Appealing features of the dataset, particularly with regard to the inclusion of earnings forecast data, include the following: (1) it is disaggregated at the firm level, thereby allowing the examination of the cross-sectional characteristics of firm-level uncertainty, (2) it contains ex-ante information on firm profitability, which is arguably better suited than ex-post information for gauging the degree of uncertainty individual firms face, and (3) the data is available in real time so that researchers and policymakers can gauge and monitor the level of uncertainty in a timely manner.

The firm-level measures of uncertainty uncover the following new facts. First, the degree of uncertainty facing individual firms differs across firms; for example, Apple's measure of uncertainty was much lower than Ford's during the Great Recession in 2009, and vice versa during the dot-com recession in 2001. Second, the first and second moments of the distribution of firm-level uncertainty measures are countercyclical. Specifically, they are negatively correlated with real GDP series.

In light of the evidence above, I propose a new model that features heterogeneous uncertainty, and I study its role in propagating aggregate shocks. My model builds on a standard heterogeneous firm business cycle model, but the model incorporates three key features of firm-level productivity dynamics distinguishing it from existing studies. First, firms face imperfect information about their productivity. While each firm has a specific productivity level, this level is not directly observable. Instead, firms receive private signals about their productivity, which they use to learn and update their beliefs over time in a Bayesian way.[4] Second, firm-level productivity changes infrequently, similar to a regime-switching process. At random intervals, a firm's productivity may be reset to a new level drawn from a known distribution. Firms are aware when these resets occur but do not immediately know the new productivity level, requiring them to restart their learning process. Third, the precision of the private signals that firms receive about their productivity is time-varying and stochastic. This signal noise is common across all firms, introducing an element of aggregate uncertainty into the model. As the noise level changes, it affects all firms' learning and ability to accurately assess their productivity simultaneously.

This paper presents an innovative mechanism that emerges from the above setting, exploring the role of noise in productivity signals as a source of uncertainty faced by individual firms. I investigate the impact of a rise in the variance of noisy signals received by firms on their

[4]Bernanke (1983) develops a single-firm, partial equilibrium model with dynamic Bayesian inference specifications to study short-term fluctuations of irreversible investment under time-varying option values.

dynamics and the aggregate economy and find that such uncertainty shocks can generate cyclical fluctuations in the aggregate economy consistent with empirical data, but through a distinct mechanism that is novel in the literature. In the model, firms are Bayesian and make investment decisions based on their beliefs about productivity, which may not necessarily align with true productivity. The key determinant of aggregate output in this setting is the cross-sectional distribution of beliefs about productivity. Bayesian firms form expectations by relying on signals, but the weight they assign to the observed signal depends on its accuracy, as measured by the variance of noise in the private signal. When signals are completely accurate, firm beliefs match the true value of productivity. However, when signals are noisy, firms place less weight on the less informative signals and more weight on the unconditional mean of productivity across firms. As a result, the dispersion of beliefs held by firms about their productivity decreases, clustering more closely around the average productivity level of all firms.

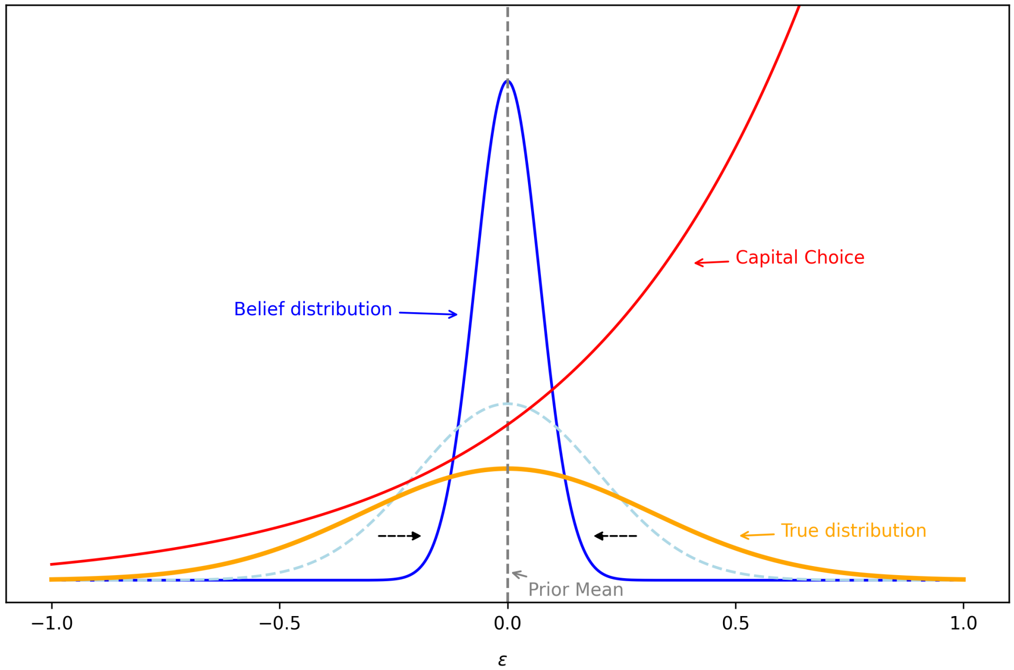
The major aggregate implications are two-fold.[5] First, a more compressed distribution of beliefs leads to a greater discrepancy between the true distribution of productivity and the belief distribution. This implies that each firm's choice of capital stock generally differs more from what the truth implies, resulting in greater misallocation of capital across firms and depressed aggregate productivity and output. Second, a more compressed distribution of beliefs leads to a lower aggregate capital stock in the economy. The key factor is the decreasing returns to scale in the production technology, a common assumption in the literature, which leads to firms' capital stock choices being convex in their conditional expectations. With such convex capital stock choices, the dispersion of expectations and beliefs across firms plays a prominent role: greater dispersion leads to higher aggregate capital stock, while more compressed beliefs lead to lower aggregate capital stock. Uncertainty shocks in the form of a rise in the variance of noise in the signal result in a more compressed distribution of firm beliefs, depressing aggregate capital and output. To demonstrate these effects, I first present a simple analytical model that captures the key mechanisms at play. I then extend the analysis to a quantitative equilibrium business cycle model and show that the insights from the analytical model carry over. This paper offers a new perspective on the sources and consequences of economic uncertainty by highlighting the importance of noisy productivity signals in shaping firm behavior and aggregate outcomes.

Related Literature The idea that links uncertainty to business cycles and especially to the slow rate of recovery after slumps dates back to Keynes (1936) and was further formulated by Bernanke (1983) in his study of investment fluctuations.[6] In the recent equilibrium business

[5] Figure 1 shows dynamics of the belief distribution (the blue and light blue lines, alongside the true productivity distribution (the yellow line) and the capital choice by firms (the red line).

[6] As stated in The General Theory, Ch. 22, *"it might be possible to achieve a recovery without the elapse of any considerable interval of time [. . .]. But, in fact, this is not usually the case [. . .]. It is the return of confidence, to speak in ordinary language, which is so insusceptible to control in an economy of individualistic capitalism. This is the aspect of the slump which bankers and business men have been right in emphasizing..."*

Figure 1: Cross-sectional Distribution of Beliefs and Capital Choice



cycle literature, the seminal contribution of Bloom (2009) studies a business cycle model in which individual firms face time-varying volatility shocks to their own productivities. He shows that uncertainty shocks, defined as a shock to the variance of the idiosyncratic productivity process, generate bust-boom cycles. A rise in stochastic volatility, in a setting where firms face nonlinear costs of factor adjustment, deters investment as firms adopt a "wait and see" policy in response to the shock. In this class of models with exogenous shocks to volatility, the aggregate effects tend to be short-lived. However, Bachmann et al. (2013) argue that the quick recovery following the wait-and-see effect is not consistent with U.S. data. In particular, they document persistent and prolonged dynamics following a rise in their measure of uncertainty. I contribute to this literature by developing a tight link between uncertainty at the start of a recession and the gradualism of the subsequent recovery.

My paper contributes to the literature that examines firm-level uncertainty using microdata and its aggregate implications by simulating a quantitative model. For example, Vavra (2014) shows that the real effect of a monetary shock decreases as the average level of uncertainty across firms increases in a price-setting model with stochastic volatility in firm-level productivity. Baley and Blanco (2018) adapt a Bayesian approach as in my paper for a price-setting model and show that the dispersion of firm-level uncertainty matters for the real effect of a monetary shock. The direction of these studies is also shared by Ilut and Saijo (2018), who investigate a business cycle amplification mechanism with ambiguity. My paper differs as it studies the dynamics of capital misallocation due to time-varying uncertainty.

Orlik and Veldkamp (2015) study an alternative origin of uncertainty fluctuations in a model of Bayesian learning. Uncertainty is associated with doubt about the true model of the economy. In particular, they argue that small increases in the awareness of tail risk is important in driving fluctuations of uncertainty.[7] In contrast, the distribution of outcomes is known to firms in my paper. What is unknown is their own actual realizations of outcomes.

In recent years, interest in uncertainty and learning over the business cycle has increased.[8] For example, Fajgelbaum et al. (2017) show a mechanism by which recessions increase uncertainty in a model of irreversible investment. Saijo (2017) builds a model with nominal rigidities and proposes a mechanism for endogenous fluctuations in uncertainty. Both papers analyze fluctuations in the amount of information available to agents. In recessions, economic activity contracts, and this reduces the flow of information and increases uncertainty. Neither this feedback nor real and nominal rigidities is necessary in my model for uncertainty shocks to produce recessions. Furthermore, unlike these papers, my model has time-varying distribution of firms, which is part of the aggregate state. Following uncertainty shocks, it delivers endogenous fluctuations in TFP through changes in the degree of misallocation of capital and labor, leading to a rapid downturn and a slow recovery in the presence of asymmetric firm decision rules, as discussed by Ilut, Kehrig, and Schneider (2017).

This work is also related to existing papers that study the role of the allocation of resources across heterogeneous agents and its impact on aggregate productivity (e.g., Restuccia and Rogerson, 2008). Hsieh and Klenow (2009) argue that misallocation of resources has a substantial impact on aggregate TFP in India and China. More recently, the role of financial frictions in generating capital misallocation and its aggregate implications have been studied in several quantitative environments (Khan and Thomas, 2013; Buera and Moll, 2015; Buera et al., 2011). Instead of financial frictions, I study the role of information frictions in causing a loss in aggregate productivity through the misallocation of resources. David et al. (2016) also study misallocation in a model of learning at the firm level. However, my paper looks at the implications of misallocation over business cycles, while they focus on a stationary equilibrium.

I also contribute to the empirical literature on uncertainty. Several proxies have been developed within the literature, ranging from the volatility of GDP or stock prices to disagreement and forecast errors in survey data, as uncertainty is difficult to identify. For example, Leahy and Whited (1996) construct a measure of uncertainty from the volatility of stock returns for individual firms. Guiso and Parigi (1999) use survey data on demand forecasts by Italian firms to infer the level of uncertainty facing these individual firms.[9] Bond et al. (2005) consider

[7]See also Kozlowski, Veldkamp and Venkateswaran (2018).

[8]There are papers that examine economic environments wherein agents learn from market outcomes. For example, Van Nieuwerburgh and Veldkamp (2006) and Caplin and Leahy (1993) study the relation between the flow of information and economic activity in models without uncertainty shocks.

[9]Guiso and Parigi (1999) use 3-point probability distributions from the Bank of Italy Survey of Investment in manufacturing (SIM) and Morikawa (2013) uses 2-point distributions from his original survey and finds that

several measures including volatility in monthly consensus earnings forecasts, the variance of forecast errors for consensus forecasts and the dispersion in earnings forecasts across market analysts. To estimate the impact of uncertainty on investment, they use panel data and look at the cross-sectional features of firm uncertainty and the investment behavior of individual firms, rather than the uncertainty distribution's cyclical properties as in Bloom et al. (2018), Kehrig (2015) and Vavra (2014). In this paper, I use data on earnings forecasts by individual analysts as in Bond et al. (2005); however, I examine not only the average cross-sectional distribution but also the cyclical changes of this uncertainty measure. Bachmann et al. (2013) use survey data from the IFO Business Climate Survey, which asks forecasters about their own future prospects rather than about macroeconomic variables such as GDP, to extensively study various measures of uncertainty. I also use forecast disagreement to measure uncertainty.

My model builds on Jovanovic's (1982) learning model, which has been applied to study a broad range of topics such as the disparate response of heterogeneous firms to aggregate shocks (Li and Weinberg, 2003; Alti, 2003) and the differential sensitivity of product switching behavior among exporters learning about their demand (Timoshenko, 2015).

The rest of the paper is organized as follows. Section 2 reports empirical results. In Section 3, the model of heterogeneous firms with learning is developed. Section 4 presents my quantitative results, both stationary equilibrium results matched against a variety of micro-level moments and the business cycle results in the presence of aggregate uncertainty. Section 5 concludes.

2 Empirics

In this section, I focus on firm-level uncertainty and show that it varies not only in the cross-section but also across time. I first build an annual panel dataset of *ex-ante* forecast dispersion and *ex-post* forecast errors on firms' earnings, using data from the I/B/E/S and Compustat. I then show that the level of the forecast-based uncertainty differs substantially across firms. Consistent with existing measures, these new measures show that the average level of uncertainty across firms is countercyclical. In particular, a sharp rise in uncertainty is observed during the Great Recession and the Covid-Pandemic Recessions. Below, I explain my measures of firm-level uncertainty, its cross-sectional features and cyclical properties.

uncertainty related to the tax system and trade policy matters for firms' capital investment and overseas activities.

2.1 Data: Measuring firm-level uncertainty

The first data source that I use is the I/B/E/S, which contains a point forecast of earnings per share (EPS) made by an individual analyst. For each firm in the data, a researcher can calculate the cross-analyst dispersion of earnings forecasts and I use it as a measure of firm-level uncertainty, *forecast dispersion.* Specifically, I construct two measures of firm-level uncertainty based on forecast dispersion. The first one is the standard deviation of earnings forecasts across analysts (= $\mathbf{Fdis^{SD}}$). I then normalize it by the average earnings forecast to obtain the second measure (= $\mathbf{Fdis^{CV}}$). Formally, this is calculated as

$$Fdis_t^{CV} = \frac{Fdis_t^{SD}}{|E_t(R_{t+1})|},$$

where $|E_t(R_{t+1})|$ denotes the absolute value of the median earnings forecast—*a consensus forecast.*

One distinctive feature of these uncertainty measures based on forecast dispersion is that they contain ex-ante information, in contrast to other uncertainty measures used in previous studies. Ex-ante information is attractive as we can elicit the level of uncertainty perceived by agents when they make decisions. It is also attractive as it allows us to directly monitor in real time the level of uncertainty faced by firms, before actual earnings outcomes are realized. Since the I/B/E/S database also contains actual earnings records, I can further calculate forecast errors from the earnings forecasts.[10]

When calculating forecast errors, I consider two different measures: Return on Assets (ROA)-based (= $\mathbf{FE^{roa}}$) and dollar-based (= $\mathbf{FE^{pct}}$) measures. $\mathbf{FE^{roa}}$ is the percentage point deviation of the realized ROA from the consensus ROA forecast, as shown below.

$$FE_t^{roa} = \frac{|(R_t - E_t(R_{t+1}))| * CSHO_t}{AT_{t-1}}$$

$CSHO_t$ denotes the number of outstanding common shares during year t, and AT_{t-1} is total assets at the beginning of year t, both taken from Compustat. Next, the dollar-based forecast error, $\mathbf{FE^{pct}}$, can be simply calculated as,

$$FE_t^{pct} = \frac{|R_t - E_t(R_{t+1})|}{|E_t(R_{t+1})|}.$$

I link the I/B/E/S and Compustat databases into a firm-by-year panel from 1976 to 2022, and the resulting dataset is an unbalanced panel of 11,938 firms, each with 6.5 data points on average. More details on variable construction are presented in Appendix. In addition to

[10]Earnings that can be obtained from the I/B/E/S are so-called street earnings, which are different from earnings that can be obtained from Compustat using the generally accepted accounting principles (GAAP).

the uncertainty measures, ($\mathbf{Fdis^{SD}}$, $\mathbf{Fdis^{CV}}$, $\mathbf{FE^{roa}}$, $\mathbf{FE^{pct}}$), my panel dataset contains firm performance measures such as ROA and other firm characteristics including size, age and analyst coverage. The top panel of Table 2 reports the summary statistics of the panel dataset, showing heterogeneity across firms in terms of performance and characteristics.

2.2 Cross-Sectional Properties: Uncertainty Varies Across Firms

In Table 2, I report the summary statistics of the uncertainty measures defined above: $\mathbf{Fdis^{SD}}$, $\mathbf{Fdis^{CV}}$, $\mathbf{FE^{pct}}$ and $\mathbf{FE^{roa}}$, all expressed in percentage (middle and bottom panels). As shown in the middle panel (rows 9 to 10), it is evident that *forecast-dispersion-based* uncertainty measures exhibit a substantial heterogeneity across firms, regardless of whether the measure is normalized or not by consensus forecasts. In the bottom panel of the table (rows 11 to 12), *forecast-error-based* uncertainty measures are reported. It follows that analysts tend to make forecast errors; at median, they under- or over-estimate EPS by about 30% ($\mathbf{FE^{pct}}$). For $\mathbf{FE^{roa}}$, the mean forecast error in terms of ROA is 5.1% while the median is 1.6%.

Table 3 shows the sample mean and the standard deviation of the key variables for the subsamples of firms distinguished by uncertainty level. Specifically, I divide the sample in 2012 by a threshold uncertainty level which is implied by the mean of $Fdis^{CV}$, and the statistics in Table 3 are separately reported by low and high uncertainty groups. Firms with low uncertainty tend to be larger in size (sales, total assets, or the number of employees), to be older and more likely to survive longer. They also have greater analyst coverages (measured by the number of analysts who report forecasts).

Together, Tables 2 and 3 clearly illustrate the observed heterogeneity of firm-level uncertainty, which remains substantial across different measures. As will be discussed below, uncertainty varies not only across firms but also over business cycles.

2.3 Cyclical Properties: Uncertainty Varies Across Time

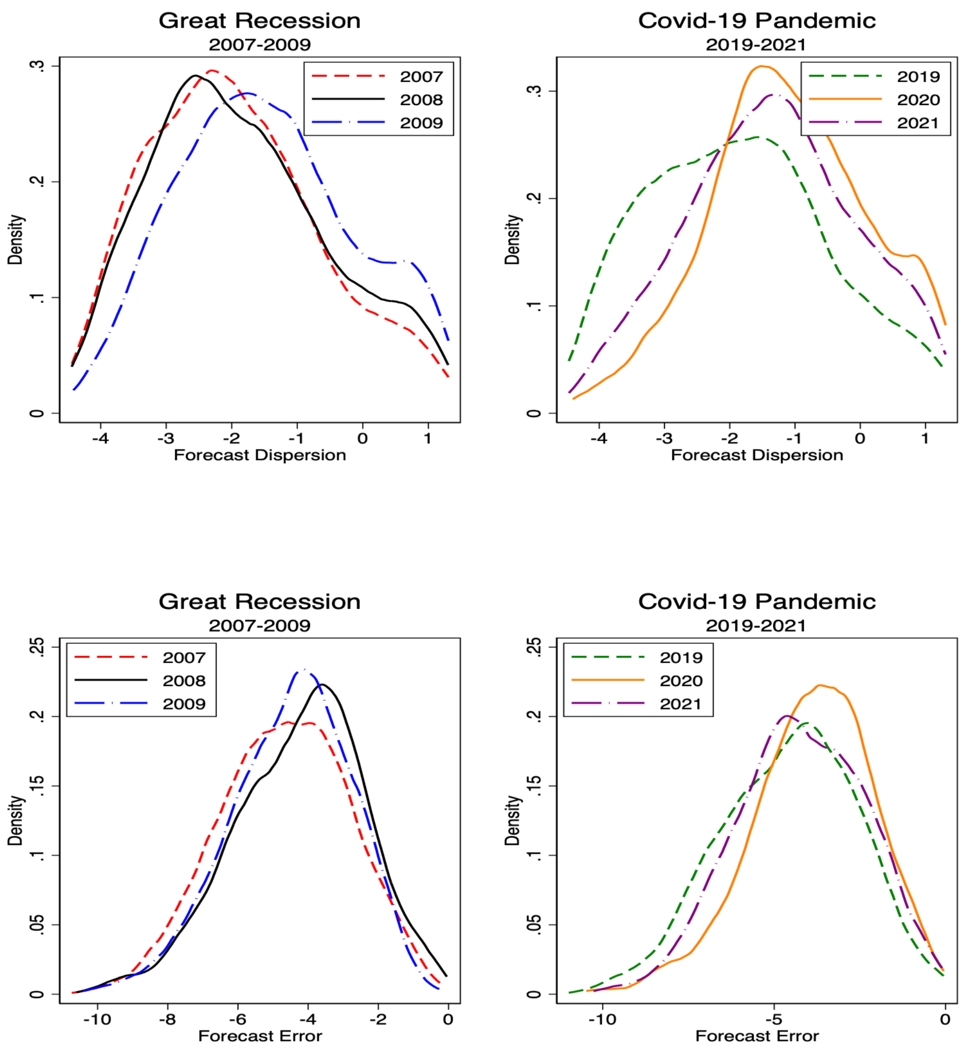
The recent development in the uncertainty literature has highlighted that macro-, industry-, and firm/establishment-level uncertainty increase in recessions (e.g. Bloom et al., 2018). Cyclical variation of uncertainty can be observed in Figures 2 and 3. Figure 2 shows how the distribution of firm-level uncertainty evolved during the Great Recession and the Covid-Pandemic recession, while the historical co-movements of uncertainty measures with GDP growth are plotted in Figure 3. In this sub-section, I investigate whether this countercyclicality holds for the uncertainty measures based on earning forecasts more formally. Moreover, by exploring how the firm-level uncertainty distribution evolves over time, I document the empirical regularity of changes in its dispersion and skewness. A set of stylized facts include: (1) the average level of firm-level uncertainty is countercyclical, (2) the dispersion of firm-level uncertainty distribution is countercyclical, and (3) the skewness of firm-level uncertainty distribution is procyclical. These results are summarized in Table 4.

In the top panel of Table 4, columns (1) and (2) report that the average level of firm-level uncertainty, measure by forecast dispersion and forecast error, are negatively correlated with GDP growth. This countercyclicality also holds for the dispersion of these uncertainty measures, as shown in columns (3) and (4). In contrast, columns (5) and (6) report a positive relationship between the skewness of the firm-level uncertainty distribution and GDP growth. As shown in the bottom panel of the table, these findings remain robust when I instead use HP-filtered GDP.[11] On the cyclicality of dispersion and skewness of the firm-level uncertainty distribution, it is noticeable that the dynamics of the distribution appears to be driven by the left tail—the low uncertainty firms. This pattern of the uncertainty distribution dynamics is also observed during the Great Recession, as in Figure 2.[12]

[11]Forecast dispersion's countercyclicality is robust to using the standard deviation instead of the coefficient of variation. See Appendix.

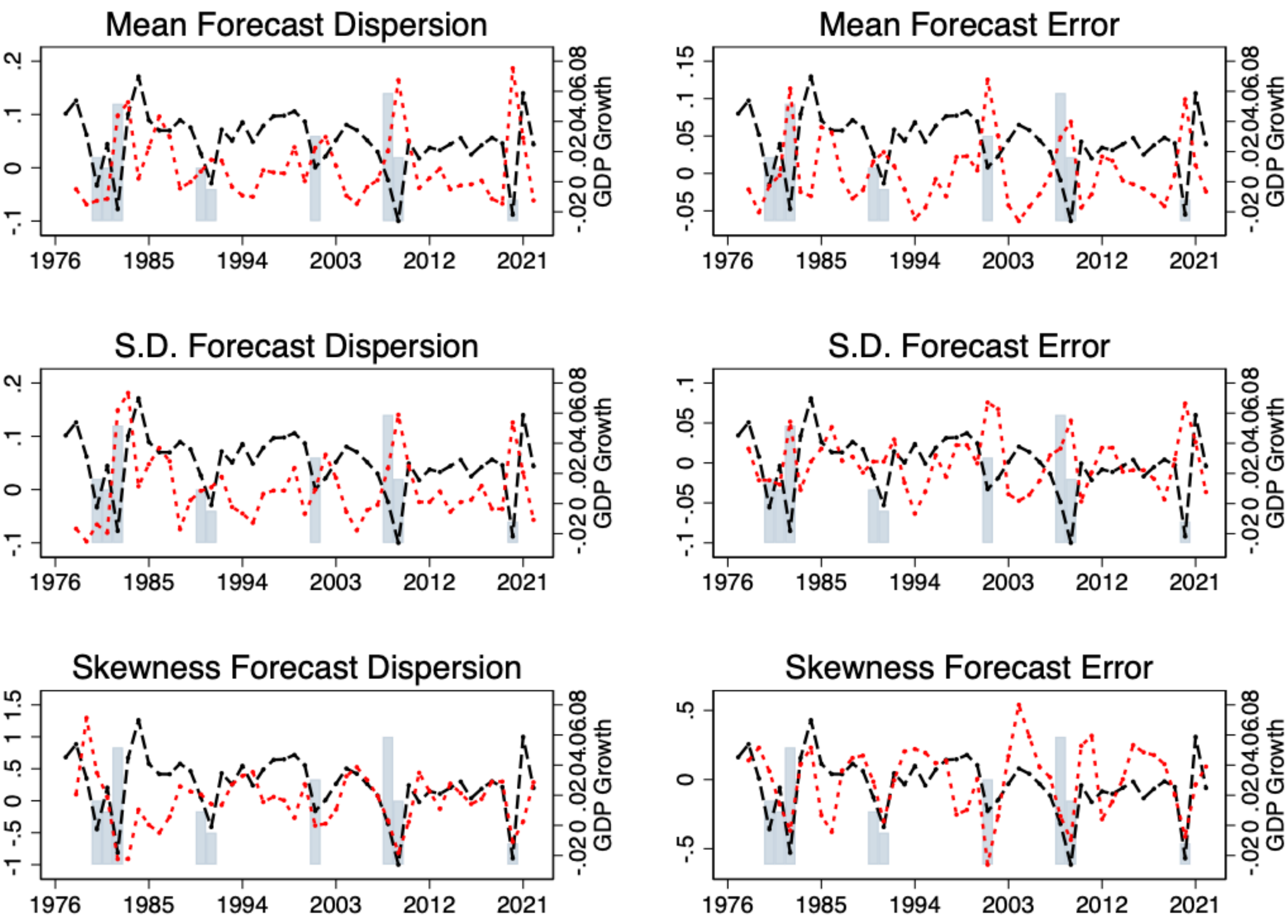
[12]This is somewhat different from the findings in other contexts such as the evolution of employment growth in Ilut, Kehrig, and Schneider (2017). There, it is that low performance firms matter, whereas firms with low uncertainty matter here and they tend to be larger in my panel data.

Figure 2: Forecast Dispersion: Great Recession vs Covid-19 Pandemic



Note: This figure compares the distributions of forecast dispersion during the Great Recession (2007-2009) and the Covid-19 Pandemic (2019-2021). The left panels show the Great Recession period with 2007 (dashed line, red), 2008 (solid line, black), and 2009 (solid line, blue). The right panels show the Covid-19 Pandemic period with 2019 (dashed line, green), 2020 (solid line, orange), and 2021 (solid line, purple). The top panel plots the distributions of the log of $\mathbf{Fdis^{cv}}$ across firms and the bottom panel plots the distributions of the log of $\mathbf{FE^{roa}}$ across firms.

Figure 3: : Historical series



Note: In the first row, the left panel shows the time-series of the cross-sectional mean of the forecast dispersion measure ($\mathbf{Fdis^{cv}}$: solid line) and the right panel shows the time-series of the cross-sectional standard deviation of the forecast error measure ($\mathbf{FE^{roa}}$: solid line). In the middle row, the left panel shows the cross-sectional mean of the forecast dispersion measure ($\mathbf{Fdis^{cv}}$: solid line) and the right panel shows the cross-sectional standard deviation of the forecast error measure ($\mathbf{FE^{roa}}$: solid line. In the bottom row, the left panel shows the cross-sectional skewness of the forecast dispersion measure ($\mathbf{Fdis^{cv}}$: solid line) and the right panel shows the skewness of the forecast error measure ($\mathbf{FE^{roa}}$: solid line). In all panels, HP-filtered real GDP series are plotted (dashed line).

3 Analytical Results

I use a simple model to analyze various effects through which uncertainty impacts aggregate output. In particular, I will present analytical results of how firms' uncertainty about their future productivity influences the firm's capital choice, the distribution of capital across firms, and aggregate output.

I consider an economy where there is a unit measure of competitive firms, each being indexed by j with production technology $y_{jt} = z_{jt}k_{jt}^{\alpha}n_{jt}^{\nu}$, where y_{jt}, z_{jt}, k_{jt}, and n_{jt} are output, productivity, capital and labor, respectively, at time t. Firm-level productivity z_{jt} is independently and identically distributed across firms with $z_{jt} = e^{\varepsilon_{jt}}$, where $\varepsilon_{jt} \sim \mathbb{N}(\bar{\varepsilon}, \sigma_{\varepsilon}^2)$. The production function is assumed to be diminishing returns to scale with $\alpha + \nu < 1$. I assume that capital physically depreciates at a constant rate δ, leading to the following capital accumulation equation, $k_{jt+1} = (1-\delta)k_{jt} + i_{jt}$, where i_{jt} is gross investment at time t. The cash flow at time t is $z_{jt}k_{jt}^{\alpha}n_{jt}^{\nu} - w_t n_{jt} - i_{jt}$, where w_t is the wage rate, taken as given by the firm. At time t, the firm observes its productivity z_{jt} and chooses its labor n_{jt} and makes its investment decision i_{jt}.

When investing in time t, I assume the firm forms beliefs about ε_{jt+1} by receiving a private signal

$$s_{jt} = \varepsilon_{jt+1} + a_{jt}, \ a_{jt} \sim \mathbb{N}(0, \sigma^2). \tag{1}$$

Lemma 1 *The firm's belief about ε_{jt+1} conditional on receiving a signal s_{jt} is as follows.*

$$\varepsilon_{jt+1} \mid s_{jt} \sim N(\tilde{\varepsilon}_{jt}, \tilde{v}) \tag{2}$$

where

$$\tilde{\varepsilon}_{jt} = \frac{\tilde{v}}{\sigma_{\varepsilon}^2}\bar{\varepsilon} + \frac{\tilde{v}}{\sigma^2}s_{jt} \tag{3}$$

$$\tilde{v} = (\frac{1}{\sigma_{\varepsilon}^2} + \frac{1}{\sigma^2})^{-1} = \frac{\sigma_{\varepsilon}^2\sigma^2}{\sigma_{\varepsilon}^2 + \sigma^2}. \tag{4}$$

Proof. See Appendix. ■

As in equation (2), the conditional distribution of ε_{jt+1} given s_{jt} is normal with mean $\tilde{\varepsilon}_{jt}$ and variance $\tilde{v}$, which are the firm's posterior mean and variance at time t, respectively. Equation (3) shows that the firm uses Bayes's rule to update its belief – that is, the posterior mean is a weighted combination of the unconditional mean and the signal, each being weighted by their relative precision. Equation (4) shows that the posterior variance $\tilde{v}$ is expressed as the inverse of the overall precision of the belief. $\tilde{v}$ is a measure of *uncertainty*, which is monotonically increasing in both σ^2 and σ_{ε}^2. Just to distinguish between them, I refer to σ^2 as *noise* and σ_{ε}^2 as *volatility* in the remaining analysis; though, it is clear that both σ^2 and σ_{ε}^2 are the drivers of $\tilde{v}$ in this setup.

Lemma 2 *The dispersion of $\tilde{\varepsilon}_{jt}$ between firms is $Var(\tilde{\varepsilon}_{jt}) = \sigma_\varepsilon^2 - \tilde{v}$, which decreases with σ^2 and increases with σ_ε^2.*

Proof. See Appendix. ■

The higher the noise in the signal received by the firms σ^2, the greater the uncertainty faced by the firms $\tilde{v}$, which compresses the distribution of the posterior mean $\tilde{\varepsilon}_{jt}$. In contrast, the distribution of the posterior mean $\tilde{\varepsilon}_{jt}$ becomes more dispersed when volatility σ_ε^2 increases, despite the fact that greater volatility also increases the uncertainty faced by the firms $\tilde{v}$. While both noise and volatility impact uncertainty in the same way, their effects on the distribution of beliefs differ, which has important implications for the distribution of capital across firms and thus aggregate output. I will examine this relationship below.

By dropping the subscript j, the firm's objective at time t is to choose labor $\{n_{t+s}\}_{s=0}^{\infty}$ and investment $\{i_{t+s}\}_{s=0}^{\infty}$ to maximize the present discounted value of cash flow, letting r and ω be the interest and wage rates, respectively, faced by the firm and assumed to be constant over time.

$$\max_{\{n_{t+s}, i_{t+s}\}_{s=0}^{\infty}} \quad E_t \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} (z_{t+s} k_{t+s}^{\alpha} n_{t+s}^{\nu} - \omega n_{t+s} - i_{t+s}) \tag{5a}$$

$$\text{s.t.} \quad k_{t+s+1} = (1-\delta) k_{t+s} + i_{t+s}, \tag{5b}$$

given an initial condition for k_t. Solving the above problem will lead to the firm's optimal choice of capital stock, and it can be aggregated across firms to yield aggregate output.

Proposition 1 *The aggregate output increases with σ_ε^2 and decreases with σ^2.*

Proof. See Appendix. ■

As shown in the proof in Appendix, aggregate output can be expressed as follows.

$$\begin{aligned} \log Y \approx\; & \underbrace{\frac{1}{1-\nu}\bar{\varepsilon} + \Big(\frac{1}{1-\nu}\Big)^2 \frac{1}{2}\sigma_\varepsilon^2}_{\text{“Jensen effect”}} + \underbrace{\frac{\alpha}{1-(\alpha+\nu)}\Big[\frac{1}{1-\nu}\bar{\varepsilon} + \Big(\frac{1}{1-\nu}\Big)^2 \frac{\tilde{v}}{2}\Big]}_{\text{“Oi-Hartman-Abel effect”}} \\ & + \underbrace{\Big[\frac{1}{2}\Big(\frac{1}{1-\nu}\Big)^2 \Big[\frac{\alpha}{1-(\alpha+\nu)}\Big]^2 + \frac{1}{(1-\nu)^2}\frac{\alpha}{1-(\alpha+\nu)}\Big]\sigma_\varepsilon^2}_{\text{“Reallocation effect”}} \\ & - \underbrace{\Big[\frac{1}{2}\Big(\frac{1}{1-\nu}\Big)^2 \Big[\frac{\alpha}{1-(\alpha+\nu)}\Big]^2 + \frac{1}{(1-\nu)^2}\frac{\alpha}{1-(\alpha+\nu)}\Big]\tilde{v}}_{\text{“Uncertainty effect”}} \end{aligned} \tag{6}$$

The first block of terms, labeled the "Jensen effect", captures how the mean and variance of the underlying productivity shocks influence output. Since I assume that $z = e^{\varepsilon}$ is distributed log-normally, an increase in σ_ε^2 increases the mean of z and thus aggregate output. I call this the "Jensen effect" following Gourio (2008).[13] The last two terms in the first line of equation (6) relate to the average level of capital stock across firms. The individual firm's optimal choice for capital stock is written as:

$$\log k_{t+1} = \frac{1-\nu}{1-(\alpha+\nu)} \log E[z_{t+1}^{\frac{1}{1-\nu}} \mid \tilde{\varepsilon}, \tilde{v}] + \text{Const.} \tag{7}$$

Since I assume $\nu < 1$, it follows $\frac{\partial \log k_{t+1}}{\partial \tilde{v}} > 0$ due to Jensen's inequality with respect to the conditional expectation. The key is that the marginal product of capital is a convex function of firm-level productivity z_t. As such, greater uncertainty increases the optimal level of capital stock at the firm level, by a mechanism similar to the Oi-Hartman-Abel effect (see Leahy and Whited (1996), Bloom (2014), and Bloom et al. (2018)).[14] Note that the Oi-Hartman-Abel effect arises not because of the assumption of decreasing returns to scale.[15] Instead, the variable labour input with the assumption of time-to-build of investment convexifies the profit function.[16]

The second and third lines together address capital allocation efficiency. The second line, which is positively scaled by σ_ε^2, suggests that greater actual productivity dispersion across firms creates greater potential output gains if capital is allocated effectively. This phenomenon, termed the reallocation effect, has an extent determined by the amount of decreasing returns to scale as has been discussed in the literature (e.g. Gilchrist and Williams (2005) and Gourio (2008)). Conversely, the third line, labeled the "Uncertainty effect" negatively scaled by $\tilde{v}$, reflects how higher uncertainty erodes these potential gains. The combined net influence of these two terms on output is proportional to $(\sigma_\varepsilon^2 - \tilde{v})$, which Lemma 2 identifies as $Var(\tilde{\varepsilon}_{jt})$—the dispersion of firms' posterior beliefs. Greater belief dispersion enhances aggregate output by improving capital allocation and thus increases aggregate output.[17] While both noise and volatility increase individual firm uncertainty ($\tilde{v}$), Lemma 2 shows their net effects on the dispersion of firms' posterior beliefs $(\sigma_\varepsilon^2 - \tilde{v})$ diverge, as do their resulting impacts on aggregate output.

An increase in volatility σ_ε^2 widens belief dispersion $Var(\tilde{\varepsilon}_{jt}) = \sigma_\varepsilon^2 - \tilde{v}$, because its direct positive effect on σ_ε^2 outweighs the associated rise in $\tilde{v}$. As such, the "Reallocation effect" term effectively dominates the "Uncertainty effect" term. This positive net impact from these two

[13] Assuming, for example, normal distribution or adjusting $\bar{\varepsilon}$ to ensure mean-preserving increases in uncertainty and volatility will eliminate the Jensen effect.

[14] This effect is also called the Hartman-Abel effect in the literature. See, for instance, Caballero (1991), Lee and Shin (2000), Bloom (2001); though, it usually refers to the effect on investment under adjustment costs, rather than the optimal level of capital stock as in this paper.

[15] In fact, Hartman (1972) and Abel (1983) assume constant returns to scale.

[16] Suppose instead that the production function can be written as $y_t = z_t k_t^\alpha$ in that capital is the only input for production. The firm's objective becomes $E_t \sum_{s=0}^{\infty} \frac{1}{(1+r)^s}(z_{t+s} k_{t+s}^\alpha - i_{t+s})$ and the optimality condition yields $\alpha E[z_{t+1}] k_{t+1}^{\alpha-1} = r + \delta$. Hence, the marginal product of capital is independent of $\tilde{v}$. See, for more detail, Caballero (1991) and Lee and Shin (2000).

[17] As seen in Appendix, those reallocation and uncertainty effects reflect the standard covariance term between productivity and capital (e.g. Olley and Pakes (1996); Bartelsman, Haltiwanger, and Scarpetta (2013)).

terms, alongside the positive influences from the "Jensen effect" and the "Oi-Hartman-Abel effect", consequently leads to an increase in aggregate output. In contrast, an increase in noise σ^2 compresses belief dispersion $Var(\tilde{\varepsilon}_{jt}) = \sigma_\varepsilon^2 - \tilde{v}$, as σ_ε^2 is unchanged while $\tilde{v}$ rises. This leads to a negative net contribution from the "Uncertainty effect" terms, with the "Reallocation effect" term unchanged. Overall, because the "Jensen effect" is not affected by an increase in noise, and the negative impact from the "Uncertainty effect" term dominates the positive "Oi-Hartman-Abel effect" term, higher noise results in a decrease in aggregate output.

The analytical results presented in this section only hold in steady state after a permanent increase in uncertainty due to noise or volatility. Moreover, the simple model is partial equilibrium, which is less suited for studying business cycles with aggregate uncertainty. I thus move to a more realistic environment with transitory shocks to noise and volatility to study whether the above results carry on. In fact, equation (6) suggests that it is unlikely that σ_ε^2 has an adverse effect on aggregate output, in contrast to the recent macroeconomic literature on uncertainty. One reason for this is that I do not assume any costs of adjusting capital stock and therefore *the real options effect* is absent in the analysis above. In the literature of uncertainty and investment, adjustment costs have been playing a prominent role in generating *the real options effect* that yields a negative relationship between uncertainty and investment.[18] As shown in the seminal papers by Bloom (2009) and Bloom et al. (2018), σ_ε^2 has a negative impact on the aggregates in the model of irreversible investment with adjustment costs, wherein firms follow the threshold investment rules which include inaction (e.g. Abel and Eberly (1995) and Abel and Eberly (1996)). More firms will undertake zero investment with greater σ_ε^2, leading aggregate investment and output to decrease. I opted not to add adjustment costs in the analysis above because adding irreversibility in the above model makes aggregation with a closed-form solution infeasible. The real option effect can amplify the impact of uncertainty on aggregate output and I will investigate this numerically in quantitative analysis below.

4 A Model of Bayesian Uncertainty Shocks

4.1 Production, Learning and Information

As in the previous section, I consider an infinite horizon model in discrete time with perfectly competitive markets and there is a unit measure of competitive firms. The production function is increasing and concave and assumed to be diminishing returns to scale as $y = zk^\alpha n^\nu$, where y, z, k, and n are output, productivity, capital and labor in each period, respectively, with

[18] There has been also an extensive discussion on the negative relationship between uncertainty and investment in the literature on irreversibility. See, for instance, Bertola (1998), Pindyck (1988), Bertola and Caballero (1994), Dixit and Pindyck (1994), among others.

$\alpha + \nu < 1$. i is gross investment in each period and the standard capital accumulation equation, $k' = (1 - \delta)k + i$, emerges with a constant depreciation rate of capital stock δ.

Firm-level productivity is persistent in that $z = e^{\varepsilon}$, where ε evolves stochastically and, as such, relevant for firms' investment decisions. In each period, ε remains with the current value with probability $1 - \pi$ or is redrawn from a distribution $N(0, \sigma_{\varepsilon}^2)$ with probability π. ε changes infrequently, and the timing of such changes, though not their value, is known to firms.

ε is only partially known to firms as it is not directly observed; however, firms can learn about ε over time by observing their private signals:

$$s = \varepsilon + a, \text{ where } a \sim \mathbb{N}(0, \sigma^2). \tag{8}$$

σ represents an exogenous variance of signal errors, which is stochastic and common across all firms: $\sigma \in \sigma_1, \ldots, \sigma_N$, where $\Pr(\sigma' = \sigma_m | \sigma = \sigma_l) = \pi_{lm} \geq 0$, and $\sum_{m=1}^{N} \pi_{lm} = 1$ for each $l = 1, \ldots, N$, the only source of aggregate uncertainty in the model.

Firms can extract information about ε by accumulating the observation of s as long as they are not hit by a reset of ε. Firms use Bayes' law to update their beliefs: the posterior mean $\tilde{\varepsilon}$ and variance $\tilde{v}$ are updated based on signals s, where $\tilde{\varepsilon}$ approaches the true value ε, while $\tilde{v}$ decreases while learning continues. On the other hand, every time ε is reset, firms start their learning afresh, leading to uncertainty cycles as in Baley and Blanco (2018) and Baley, Figueiredo, and Ulbricht (2022).

4.2 Aggregate State

In addition to σ, which evolves stochastically over time, a non-trivial, time-varying distribution of firms is a part of the aggregate state in this model economy. Firms at the beginning of each period are identified by the mean $\tilde{\varepsilon}$ and variance $\tilde{v}$ of their beliefs about their productivity, together with the current productivity ε and predetermined capital stock k. Thus, I summarize the distribution of firms over $(\tilde{\varepsilon}, \tilde{v}, \varepsilon, k)$ using the probability measure μ defined on the Borel algebra, $\mathcal{S}$, generated by the open subsets of the product space, $\mathcal{S} = \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+$. Given the distribution of firms, the aggregate state of the economy is fully summarized by (σ, μ), and the distribution of firms evolves over time according to a mapping, Γ, from the current aggregate state; $\mu' = \Gamma(\sigma, \mu)$.

4.3 Firm Problem

I formulate a firm's problem recursively, given the aggregate state (σ, μ) and thus $\mu' = \Gamma(\sigma, \mu)$. The problem consists of choosing the capital stock for the following period, k', and the current labor input, n. Let $v(\tilde{\varepsilon}, \tilde{v}, s, \varepsilon, k; \sigma, \mu)$ be the value function of a firm:

$$\begin{aligned} v(\tilde{\varepsilon},\tilde{v},s,k;\sigma,\mu) &= \max_{n,k'} \mathbb{E}_{\varepsilon|(\tilde{\varepsilon},\tilde{v},s)} \Big[e^{\varepsilon} k^{\alpha} n^{\nu} - \omega n + (1-\delta)k - k' \\ & \quad + (1-\pi)\, \mathbb{E}_{\sigma'|\sigma}\, d\left(\sigma',\sigma,\mu\right) v_0(\tilde{\varepsilon}',\tilde{v}',k';\sigma',\mu') \\ & \quad + \pi\, \mathbb{E}_{\sigma'|\sigma}\, d\left(\sigma',\sigma,\mu\right) v_0(\tilde{\varepsilon}_0,\tilde{v}_0,k';\sigma',\mu') \Big] \end{aligned} \tag{9}$$

I explain the above firm's problem line-by-line as follows. Each firm's profits are its output less wage payments and investment, as shown in the first line. With probability $1-\pi$, in the second line, the current productivity is maintained at the beginning of the next period, and hence their expectation over ε' and thus s' are conditional on $(\tilde{\varepsilon},\tilde{v},s,\sigma)$. Furthermore, they discount next period's value by the state contingent discount factor, $d\left(\sigma',\sigma,\mu\right)$. The state contingent discount factor is consistent with households decision rules in equilibrium, as will be discussed later. With probability π, in contrast, the current productivity is lost and a new one is drawn, independent of the current state. $\tilde{\varepsilon}_0$ is the prior mean, which is the unconditional mean of the distribution and thus $\tilde{\varepsilon}_0 = 0$. $\tilde{v}_0$ is the prior variance and thus $\tilde{v}_0 = \sigma_{\varepsilon}^2$.

$v_0(\tilde{\varepsilon},\tilde{v},k;\sigma,\mu)$ is the beginning-of-period expected value of a firm before it observes its private signal. Conditional on its prior belief $(\tilde{\varepsilon},\tilde{v})$, each firms takes expectations over a possible draw of private signals s:

$$v_0(\tilde{\varepsilon},\tilde{v},k;\sigma,\mu) = \mathbb{E}_{s|(\tilde{\varepsilon},\tilde{v})}\, v(\tilde{\varepsilon},\tilde{v},s,k;\sigma,\mu). \tag{10}$$

4.4 Households

There is a unit measure of identical households in this economy. Households earn labor income by supplying a fraction of time endowment in each period, and hold their wealth as a comprehensive portfolio of assets; firm shares of measure m_e and non-contingent discount bonds m_b. Period utility is given by $U(C, 1-N)$, and β represents the subjective discount factor. The representative household maximizes the lifetime expected discounted utility, V^h, by choosing the quantities of aggregate consumption, C, and labor supply, N, while adjusting its asset portfolio over time. To simplify notation in the following, I use $x \equiv (\tilde{\varepsilon},\tilde{v},s,k)$ to summarize the firm individual state.

$$V^h(m_e, m_b; s, \mu) = \max_{C,N,m_e',m_b'} \Big[U(C, 1-N) + \beta \mathbb{E}_{s'|s} V^h(m_e', m_b'; s', \mu') \Big] \tag{11}$$

$$\begin{aligned} \text{subject to :} & \\ & C + q(s,\mu) m_b' + \int_{\mathcal{S}} \rho_1(x'; s, \mu) \cdot m_e'(d[\tilde{\varepsilon} \times \tilde{v} \times s \times \varepsilon \times k]) \\ & \quad \leq \omega(s,\mu) N + m_b + \int_{\mathcal{S}} \rho_0(x; s, \mu) \cdot m_e(d[\tilde{\varepsilon} \times \tilde{v} \times s \times \varepsilon \times k]) \qquad (12) \\ & \text{and :} \quad \mu' = \Gamma(s, \mu) \qquad (13) \end{aligned}$$

Given the aggregate state, $q(s, \mu)$ is the discount bond price in the above problem. Regarding the prices of firm shares, $\rho_1(x'; s, \mu)$ denotes the dividend-exclusive prices in the current period, and $\rho_0(x; s, \mu)$ is the dividend-inclusive value of current shareholding m_e.

To define the competitive equilibrium of the model, let $C^h(m_e, m_b; s, \mu)$ and $N^h(m_e, m_b; s, \mu)$ represent the household decision rules for consumption and labor supply. Also, let $M_b^h(m_e, m_b; s, \mu)$ be the decision rule for bond holding, and $M_e^h(m_e, m_b, x'; s, \mu)$ be the choice of shares over the firm distribution with x'.

4.5 Recursive Equilibrium

In the following, I define recursive competitive equilibrium of the model while suppressing the arguments of functions for simplicity.

A recursive competitive equilibrium is a set of functions,

$$\begin{aligned} \text{prices} &: \quad (\omega, d, q, \rho_0, \rho_1) \\ \text{quantities} &: \quad (N, K, C^h, N^h, M_b^h, M_e^h) \\ \text{values} &: \quad (V, V^h), \end{aligned}$$

that solve firm and household problems and clear the markets for assets, labor, and output:

1. $\{v_0, v\}$ satisfies (9) - (10), and (N, K) are the associated policy functions for firms.
2. V^h satisfies (11), and (C^h, N^h, M_b^h, M_e^h) are the associated policy functions for households.
3. $M_e^h(m_e, m_b, x; s, \mu) = \mu(x)$ for each $x \equiv (\tilde{\varepsilon} \times \tilde{v} \times \varepsilon \times k) \in \mathcal{S}$.
4. The labor and goods markets clear.

$$
\begin{aligned}
N^h(m_e, m_b; \sigma, \mu) &= \int_{\mathcal{S}} N(x; s, \mu) \cdot \mu(d[\tilde{\varepsilon} \times \tilde{v} \times s \times \varepsilon \times k]) \\
C^h(m_e, m_b; \sigma, \mu) &= \int_{\mathcal{S}} \big[e^{\varepsilon} k^{\alpha} N(x; \sigma, \mu)^{\nu} \\
&\qquad - \big(K(x; \sigma, \mu) - (1-\delta)k \big) \big] \cdot \mu(d[\tilde{\varepsilon} \times \tilde{v} \times \varepsilon \times k])
\end{aligned}
$$

5. $\Gamma(\sigma, \mu)$ is generated by $K(\tilde{\varepsilon}, \tilde{v}, \varepsilon, k; \sigma, \mu)$, and the exogenous stochastic evolution of σ and the evolution of firms' belief based on the Baye's rule, along with the aggregation of firms' optimal choices given current state variables. The evolution of the firm distribution is defined as follows. I define the indicator function $\chi(x) = 1$ for $x = 0$ and $\chi(x) = 0$ for $x \neq 0$. I also define the probability of drawing ε' when ε is reset by $F(\varepsilon'|\sigma)$. I then define Γ as,

$$
\begin{aligned}
&\mu'(\tilde{\varepsilon}', \tilde{v}', \varepsilon, k') \\
&\qquad = (1-\pi) \int_{\mathcal{S}} \chi(k' - K(\tilde{\varepsilon}, \tilde{v}, \varepsilon, k; s, \mu)) \mu(d\tilde{\varepsilon} \times d\tilde{v} \times d\varepsilon \times dk).
\end{aligned}
$$

$$
\begin{aligned}
&\mu'(\tilde{\varepsilon}', \tilde{v}', \varepsilon', k') \\
&\qquad = \pi \int_{\mathcal{S}} F(\varepsilon') \chi(k' - K(\tilde{\varepsilon}, \tilde{v}, \varepsilon, k; s, \mu)) \mu(d\tilde{\varepsilon} \times d\tilde{v} \times d\varepsilon \times dk).
\end{aligned}
$$

Using $C(\sigma, \mu)$ and $N(\sigma, \mu)$ to describe the market-clearing values of household consumption and hours worked, it is straightforward to show that market-clearing requires that (a) the real wage equal the household marginal rate of substitution between leisure and consumption:

$$
w(\sigma, \mu) = \frac{D_2 U\Big(C(\sigma, \mu), 1 - N(\sigma, \mu) \Big)}{D_1 U\Big(C(\sigma, \mu), 1 - N(\sigma, \mu) \Big)},
$$

that (b) firms' state-contingent discount factors are consistent with the household marginal rate of substitution between consumption across states:

$$
d\left(\sigma', \sigma, \mu\right) = \frac{\beta D_1 U\Big(C(\sigma', \mu'), 1 - N(\sigma', \mu') \Big)}{D_1 U\Big(C(\sigma, \mu), 1 - N(\sigma, \mu) \Big)}.
$$

5 Quantitative Analysis

5.1 Parametrization

(1) Common parameters:

I assume that the representative household's period utility is $u(c, l) = \log c + \eta l$, with $\eta > 0$ as in models of indivisible labor (e.g., Hansen (1985); Rogerson (1988)) and c and l are consumption and leisure in each period, respectively. As seen in the previous sections, I assume that each heterogeneous firm undertakes production via the Cobb-Douglas production function: $y = e^{\varepsilon} k^{\alpha} n^{\nu}$.

Following the approach of Bloom et al. (2018), I calibrate five key parameters to match aggregate moments in the U.S. economy at an annual frequency. The discount factor β is set to 0.96, targeting a risk-free real rate of 4%. α is calibrated to 0.25, corresponding to an isoelastic demand with a markup of 33%. To achieve capital and labor shares of 1/3 and 2/3 respectively, ν is set to 0.50. The depreciation rate δ is calibrated to 0.10, matching the annual depreciation of capital stock. Finally, the labor supply parameter η is set to 2.00, targeting average hours worked of 1/3.

(2) Micro parameters:

The calibration of the micro parameters utilizes the data set described above in this paper (the merged Compustat-IBES). The parameters are chosen to reflect the persistence and variability of firm-level productivity, as well as the transition probabilities between different states of productivity. The process for ε_{t+1} is defined as:

$$\varepsilon_{t+1} = \begin{cases} \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_{\varepsilon}^2) & \text{with probability } \pi, \\ \varepsilon_t & \text{with probability } (1-\pi), \end{cases}$$

where $s = \varepsilon + a$, with $a \sim \mathcal{N}(0, \sigma^2)$, and $\sigma \in \{\sigma_L, \sigma_H\}$. The transition probabilities for σ are given by:

$$\begin{pmatrix} \rho_L & 1-\rho_L \\ 1-\rho_H & \rho_H \end{pmatrix}.$$

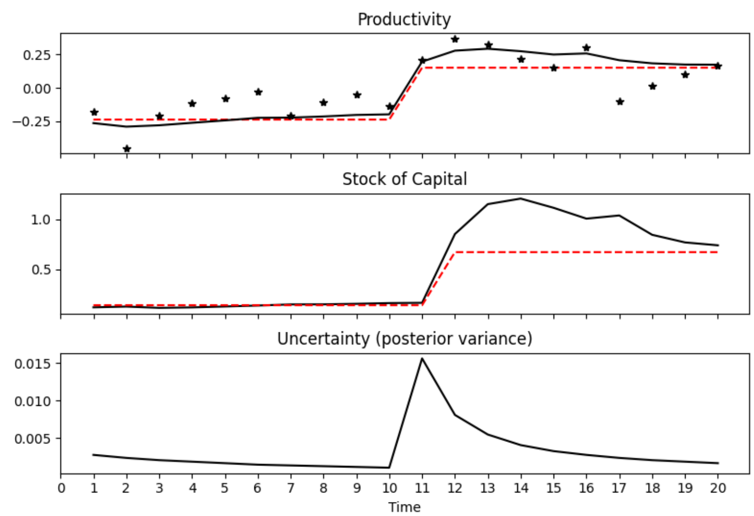
π and σ_{ε} are set to reproduce the yearly persistence of firm-level productivity of 0.95 and the cross-sectional standard deviation of 0.45. We set σ_L to 0.28, representing the cross-sectional average of $\tilde{v}/\sigma_{\varepsilon}^2 = 0.41$. σ_H is set to 0.59 to obtain $\frac{\sigma_H}{\sigma_L} = 2.1$. The yearly transition probabilities, p_L and p_H, are both set to 0.70, encapsulating both the persistence in productivity levels and the dynamics of transitions between different uncertainty regimes. Tables 5 and 6 summarize the calibrated parameters, their descriptions, and their values:

5.2 Micro-level Firm Behavior

Imperfect information about total factor productivity across firms causes a misallocation of capital and labor. Firms operating with imperfect information deviate from the optimal allocation of resources and exhibit both over- as well as undercapacity. This pattern of misallocation is distinct from that which appears with financial frictions such as lending that is subject to default risk (e.g., Khan, Senga, and Thomas (2016)) or a collateral constraint (e.g., Buera and Moll (2015)).

Figure 4 presents a simulation of the learning and capital accumulation dynamics of an individual firm. The key event in the firm's lifecycle is a resetting of its productivity that occurs at period 11, as evidenced by the abrupt upward shift in the dashed line in the top panel. As a consequence of this productivity reset, the firm experiences a sudden increase in uncertainty regarding its own productivity, as illustrated by the spike in the dashed line in the bottom panel.

Figure 4: Learning cycles from simulation



Note: This figure plots the patterns of the behavior of firms in the simulation without aggregate shocks. 1,000 firms are simulated for 200 periods, and a 20-period simulation result for one firm is shown here. The top panel shows a series of firm productivity (dashed line), signals (star dots), and the posterior belief about it (solid line). The middle panel shows a series of capital stocks (solid line). The dashed line corresponds to the frictionless level of capital stock under perfect information. The bottom panel shows a series of the conditional variance of forecasts of productivity.

The firm's uncertainty gradually diminishes over subsequent periods as it learns about its new productivity level, reflected in the downward slope of the dashed line post period 11. The firm's capital investment decisions are significantly influenced by the productivity reset. The middle panel reveals a substantial increase in the firm's capital stock, represented by the solid line, immediately following the productivity reset at period 11. However, this investment is based on signals received during these periods that exceed the true value of the firm's updated productivity. Consequently, the firm overinvests, resulting in an excessive capital stock relative to its actual productivity. As the firm gradually resolves uncertainty about its new productivity level in the periods following the reset, it correspondingly adjusts its capital stock downwards. By approximately period 20, the firm's capital stock converges to its "frictionless" level, aligning with its true productivity. This simulation underscores how a sudden shift in a firm's productivity can lead to capital misallocation that persists due to learning frictions. Each time productivities are reset, firms undergo a process of gradually adjusting their capital stocks as they resolve uncertainty about their new productivity levels. This incremental adjustment process results in periods of capital misallocation until firms' knowledge aligns with the new productivity reality. The simulation thus illustrates the interplay between productivity shocks, learning, uncertainty, and capital investment dynamics at the firm level.

5.3 Business Cycle Analysis

5.3.1 Business Cycle Statistics

We now examine the business cycle implications of our model with Bayesian learning and time-varying uncertainty. Table 1 presents the key business cycle moments from an unconditional simulation of our model at annual frequency, comparing them with U.S. data from 1976 to 2014.

The model successfully generates business cycle fluctuations of realistic magnitude. Output volatility in the model is 0.347%, which represents about 18% of the empirical volatility of 1.940%. While the model generates lower absolute volatility than in the data, it captures the key qualitative features of business cycle dynamics.

Most importantly, the model reproduces the standard business cycle regularities. Consumption, investment, and hours all move procyclically with output, exhibiting positive contemporaneous correlations of 0.388, 0.902, and 0.870, respectively. These correlations are broadly consistent with their empirical counterparts (0.915, 0.843, and 0.847).

The model also captures the relative volatilities of key macroeconomic variables. Investment is substantially more volatile than output, with a relative standard deviation of 7.208 compared to 4.440 in the data. This heightened investment volatility reflects firms' dynamic responses to uncertainty about their productivity, as they adjust their capital stocks based on evolving beliefs. Consumption is less volatile than output (relative volatility of 0.496 versus 0.904 in the data), consistent with households' desire to smooth consumption over time. Hours worked exhibits intermediate volatility (0.902 times output volatility), somewhat lower than the empirical value of 1.336.

A notable feature of our model is that these business cycle dynamics emerge entirely from time-varying uncertainty in firms' signals about their productivity. Unlike standard RBC models that rely on aggregate TFP shocks, or the stochastic volatility models in the uncertainty literature, our fluctuations arise from changes in the precision of information that firms receive. When signals become noisier (higher σ), firms' beliefs about their productivity become more compressed around the unconditional mean, leading to capital misallocation and reduced aggregate output.

This mechanism generates endogenous persistence in economic activity. Following an uncertainty shock, firms gradually update their beliefs as they receive new signals, creating a slow recovery in productivity and output. This contrasts with models featuring exogenous volatility shocks, where the recovery tends to be more rapid once uncertainty subsides.

Table 1: : Unconditional business cycle moments

	US Data, 1976 to 2014			**Model Simulation**		
x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$Corr(x,Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$Corr(x,Y)$
Y	1.940	1.000	1.000	0.347	1.000	1.000
C	1.850	0.904	0.915	0.172	0.496	0.388
I	8.537	4.440	0.843	2.499	7.208	0.902
N	2.592	1.336	0.847	0.322	0.902	0.870

Note: The above table shows the data and model business cycle moments of output Y, consumption C, investment I, and hours worked N. $\sigma(x)$ is the standard deviation of x, and $\sigma(x)/\sigma(Y)$ is the relative standard deviation to that of Y, and $Corr(x,Y)$ is the contemporaneous correlation of x with Y. The model moments are obtained from a 1,000-period unconditional simulation using the solution of the model. All series are HP-filtered in logs with a smoothing parameter of 100. The data used to generate the above moments are: (1) real gross domestic product (GDPCA taken from FRED), (2) investment is real gross private domestic investment (GPDICA taken from FRED), (3) consumption is real personal consumption expenditures (DPCERX1A020NBEA taken from FRED), and (4) hours is total nonfarm business sector hours (HOANBS taken from FRED but anuualized).

5.3.2 Aggregate Dynamics Following a Bayesian Uncertainty Shock

To understand the aggregate implications of uncertainty shocks in our framework, we examine how the economy responds when firms suddenly face noisier signals about their productivity. Figure 5 presents the dynamic responses of key macroeconomic variables to a Bayesian uncertainty shock—specifically, an increase in σ^2 that makes firms' private signals less informative. The responses are computed by independently simulating 2,000 economies for 60 years each. An uncertainty shock is introduced in year 31 by imposing a high uncertainty state, and we track the average percent deviation from pre-shock levels across all simulations.[19]

When the uncertainty shock hits in period 1, firms' belief updating becomes less responsive to their private signals. With σ^2 being elevated, the signal-to-noise ratio deteriorates, causing firms to place less weight on their noisy signals (s) and more weight on their prior belief ($\tilde{\varepsilon}$) when forming their posterior belief ($\tilde{\varepsilon}'$).[20]

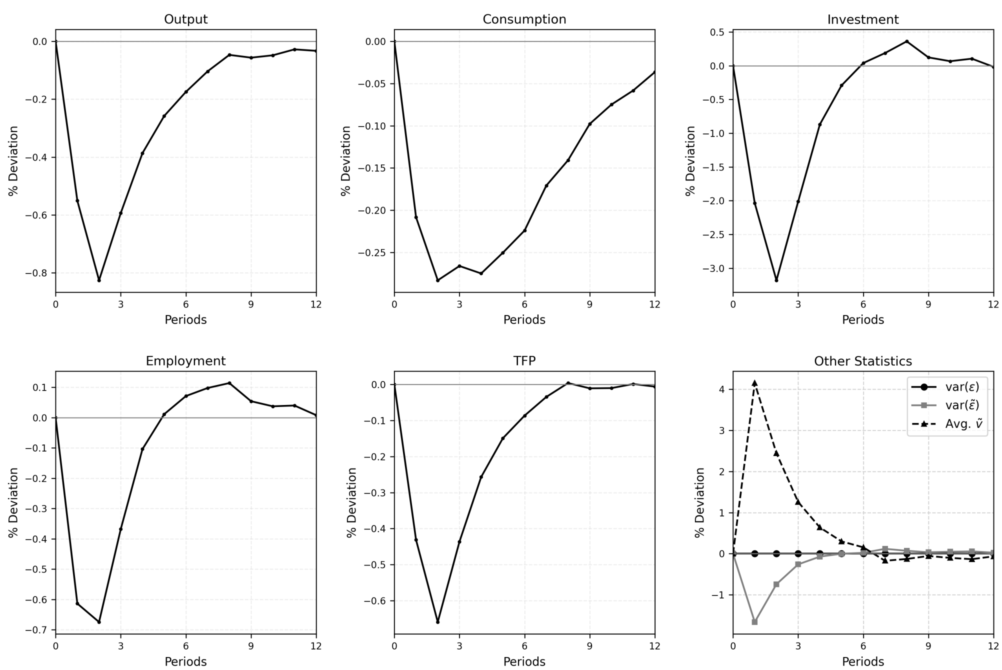
[19] In each simulation, the first 30 years evolve unconditionally. For presentation clarity, Figure 5 relabels time periods: year 30 (pre-shock) is shown as period 0, year 31 (shock impact) as period 1, and so forth.

[20] From equation (3) in Section 3, the Bayesian updating rule is $\tilde{\varepsilon}' = \frac{\tilde{v}}{\sigma_{\varepsilon}^2}\bar{\varepsilon} + \frac{\tilde{v}}{\sigma^2}s$, where $\tilde{v} = \frac{\sigma_{\varepsilon}^2\sigma^2}{\sigma_{\varepsilon}^2+\sigma^2}$ and $\bar{\varepsilon}$ denotes the unconditional mean. When σ^2 increases, the weight on the signal $\frac{\tilde{v}}{\sigma^2} = \frac{\sigma_{\varepsilon}^2}{\sigma_{\varepsilon}^2+\sigma^2}$ decreases.

This reduced reliance on private signals has two important consequences. First, while learning from signals always reduces individual uncertainty ($\tilde{v} > \tilde{v}'$), the magnitude of this reduction is smaller when signals are noisier.[21] Therefore, the average posterior variance $\tilde{v}'$ is higher in period 1 when the uncertainty shock hits than it would have been without the shock. This elevated uncertainty following the shock is visible in the bottom right panel of Figure 5.

Second, with less weight placed on heterogeneous private signals, firms' posterior beliefs $\tilde{\varepsilon}'$ become more compressed around the unconditional mean $\bar{\varepsilon}$ compared to their prior beliefs $\tilde{\varepsilon}$. That is, the cross-sectional variance of beliefs decreases: $\mathrm{Var}(\tilde{\varepsilon}) > \mathrm{Var}(\tilde{\varepsilon}')$. This compression of beliefs can also be seen in the bottom right panel of Figure 5. Production and investment decisions must then be made based on these more homogeneous, and therefore less informative, beliefs.

Figure 5: : Aggregate Dynamics Following a Bayesian Uncertainty Shock



Note: Each panel plots the aggregate economy's response to an uncertainty shock, which hits the economy in period 1. The percent deviation from the variable's pre-recession average level is shown.

Let us first examine investment decisions. Proposition 1 helps us understand how the uncertainty shock affects investment through two opposing forces. With $\tilde{v}'$ being larger on average across firms due to noisier signals, the Oi-Hartman-Abel effect pushes investment upward—since each firm's optimal capital choice is convex in productivity, higher uncertainty about future pro-

[21] From equation (4), when a firm with prior variance $\tilde{v}$ receives a signal with noise σ^2, the posterior variance becomes $\tilde{v}' = \left(\frac{1}{\tilde{v}} + \frac{1}{\sigma^2}\right)^{-1} = \frac{\tilde{v}\sigma^2}{\tilde{v}+\sigma^2}$. The variance reduction is thus $\tilde{v} - \tilde{v}' = \frac{\tilde{v}^2}{\tilde{v}+\sigma^2}$, which decreases as σ^2 increases.

ductivity leads firms to choose larger capital stocks. However, the uncertainty effect works in the opposite direction and dominates. With $\mathrm{Var}(\tilde{\varepsilon}') < \mathrm{Var}(\tilde{\varepsilon})$ due to belief compression, the aggregate capital stock based on the more compressed distribution of $\tilde{\varepsilon}'$ is smaller than it would be with the more dispersed prior distribution. Additionally, this compression creates capital misallocation: productive firms (with high true ε) have posterior beliefs pulled toward the mean and therefore choose too little capital, while unproductive firms have beliefs pushed up toward the mean and choose too much. These two components of the uncertainty effect—reduced aggregate capital and increased misallocation—together dominate the Oi-Hartman-Abel effect, causing investment to fall.

Production decisions through labor choices follow a similar mechanism. Labor is chosen within the period after observing signals and updating beliefs. With $\tilde{v}'$ being larger on average across firms due to noisier signals, the Oi-Hartman-Abel effect pushes labor demand upward—since each firm's optimal labor choice is convex in productivity, higher uncertainty leads firms to demand more labor. However, as with investment, the uncertainty effect dominates. With firms' posterior beliefs compressed around the mean (i.e., $\mathrm{Var}(\tilde{\varepsilon}') < \mathrm{Var}(\tilde{\varepsilon})$), the aggregate labor demand based on the more compressed distribution of $\tilde{\varepsilon}'$ is smaller than it would be with the more dispersed prior distribution. Moreover, labor is misallocated across firms: productive firms (with high true ε) have posterior beliefs pulled toward the mean and therefore hire too little, while unproductive firms have beliefs pushed up toward the mean and hire too much. These two components of the uncertainty effect—reduced aggregate labor and increased misallocation—cause employment to fall in the impact period.

The resulting output decline reflects both the direct effect of reduced factor inputs and the efficiency loss from their misallocation. With compressed beliefs, productive firms underestimate their true productivity and employ too few resources (both labor today and capital for tomorrow), while unproductive firms overestimate their productivity and employ too many. This misallocation manifests as a decline in total factor productivity—the economy produces less not only because it uses fewer inputs in aggregate, but because it allocates those inputs inefficiently across firms. The TFP panel in Figure 5 confirms this efficiency loss, showing a persistent decline that reflects the ongoing misallocation as firms slowly learn their true productivity and gradually reallocate resources.

With both labor and TFP falling, output contracts immediately, and consumption also decline from the outset.[22] This immediate co-movement of consumption and output, driven by the direct effect of compressed beliefs on production decisions, represents a key contribution of our Bayesian uncertainty framework to understanding business cycle dynamics.

[22]This contrasts sharply with models like Bloom et al. (2018) where adjustment costs and the "wait-and-see" effect can generate an initial consumption rise as firms pause investment, temporarily freeing up resources. In our framework, the immediate output contraction through reduced labor demand and misallocation dominates any resource reallocation from lower investment. Consequently, there is no consumption "puzzle"—consumption falls alongside output from the shock's onset, though less sharply due to households' consumption smoothing motives.

A defining feature of our model is the asymmetry between the sharp initial decline and gradual recovery. The fall is immediate because the uncertainty shock instantly affects all firms' signals in period 1, leading to compressed beliefs, misallocated inputs, and lower output within the same period.

The recovery, however, is remarkably gradual. Even after σ^2 returns to normal, belief compression persists. Each firm must slowly rebuild its knowledge about its true productivity through sequential signal observations. Productive firms need multiple good signals before their beliefs realign with their true productivity, while unproductive firms need consistently poor signals to correct their overestimation. This learning process is inherently slow, constrained by signal noise and Bayesian updating.

This generates the persistent dynamics visible in Figure 5. Output, investment, and employment remain depressed for many periods—not because uncertainty remains high, but because the distribution of beliefs takes time to decompress back to its efficient dispersion. The gradual TFP recovery particularly reflects this progressive resolution of misallocation as resources slowly reallocate toward productive firms. This persistence, arising endogenously from information frictions rather than adjustment costs or persistent shocks, distinguishes our Bayesian uncertainty framework from existing models.

The persistence and magnitude of these contractionary effects highlight the importance of belief dynamics in propagating uncertainty shocks through the economy. While we have focused on our baseline calibration here, Appendix D confirms these patterns are robust across a wide range of parameter values. Additionally, Appendix E provides a parallel analysis of volatility shocks (σ_z^2), demonstrating how increased productivity dispersion affects aggregate dynamics.

6 Conclusion

I construct ex-ante measures of uncertainty for each individual firms using forecast dispersion among market analysts. I argue that my ex-ante measure is appealing in capturing uncertainty perceived by agents. It is also attractive as my measure is available in real time at the firm level so that we can use it to gauge the level of uncertainty in a timely manner. Using my measure, I show the level of uncertainty faced by firms varies both in the cross-section and in the time series.

I then propose a new approach to modelling uncertainty in a Bayesian way. By using a simple theoretical framework, I show that uncertainty in a Bayesian sense operates differently from uncertainty in a stochastic volatility sense. I demonstrate that a greater uncertainty in a Bayesian sense leads to a more compressed belief distribution across firms, which, in turn, decreases the aggregate productivity and capital stock without relying on any adjustment costs of capital or investment.

These theoretical mechanisms are quantitatively investigated in a more fully-fledged equilibrium business cycle model and the quantitative exercises highlight a prominent role of uncertainty shocks in driving aggregate fluctuations in a business cycle frequencies and the nature of aggregate dynamics is consistent with what can be seen in data for the US economy.

References

Abel, Andrew B. 1983. "Optimal Investment Under Uncertainty." *The American Economic Review* 73 (1):228–233.

Abel, Andrew B and Janice C Eberly. 1995. "A Unified Model of Investment Under Uncertainty." *American Economic Review* .

Abel, Andrew B. and Janice C. Eberly. 1996. "Optimal Investment with Costly Reversibility." *The Review of Economic Studies* 63 (4):581–593.

Albagli, E., C. Hellwig, and A. Tsyvinski. 2015. "A Theory of Asset Prices Based on Heterogeneous Information." .

Alti, A. 2003. "How Sensitive Is Investment to Cash Flow When Financing Is Frictionless?" *Journal of Finance* 58 (2):707–722.

Arellano, C., Y. Bai, and P. J. Kehoe. 2018. "Financial Frictions and Fluctuations in Volatility." *Journal of Political Economy* forthcoming.

Bachmann, R. and C. Bayer. 2013. "'Wait-and-See' Business Cycles?" *Journal of Monetary Economics* 60 (6):704–719.

Bachmann, R., S. Elstner, and E. R. Sims. 2013. "Uncertainty and Economic Activity: Evidence from Business Survey Data." *American Economic Journal: Macroeconomics* 5 (2):217–249.

Baker, Scott R., Nicholas Bloom, and Stephen J. Terry. 2020. "Using Disasters to Estimate the Impact of Uncertainty." *NBER Working Paper Series* .

Baley, I. and J. A. Blanco. 2018. "Firm Uncertainty Cycles and the Propagation of Nominal Shocks." *American Economic Journal: Macroeconomics* forthcoming.

Baley, Isaac, Ana Figueiredo, and Robert Ulbricht. 2022. "Mismatch Cycles." *Journal of Political Economy* 130 (11):2943–2984.

Bartelsman, E.J, J Haltiwanger, and S Scarpetta. 2013. "Cross-Country Differences in Productivity: The Role of Allocation and Selection." *The American Economic Review* 103 (1):305–334.

Basu, S. and B. Bundick. 2017. "Uncertainty Shocks in a Model of Effective Demand." *Econometrica* 85 (3):937–958.

Bentolila, Samuel and Giuseppe Bertola. 1990. "Firing Costs and Labour Demand: How Bad is Eurosclerosis?" *The Review of Economic Studies* 57 (3):381–402.

Bernanke, B. S. 1983. "Irreversibility, Uncertainty, and Cyclical Investment." *Quarterly Journal of Economics* 98 (1):85–106.

Bertola, GIUSEPPE. 1998. "Irreversible investment." *Research in Economics* 52 (1):3–37.

Bertola, Guiseppe and Ricardo J. Caballero. 1994. "Irreversibility and Aggregate Investment." *The Review of Economic Studies* 61 (2):223–246.

Blanchard, O. 2009. "(Nearly) nothing to fear but fear itself." *The Economist* .

Bloom, N., S. Bond, and J. Van Reenen. 2007. "Uncertainty and Investment Dynamics." *Review of Economic Studies* 74 (2):391–415.

Bloom, N., S. J. Davis, L. Foster, B. Lucking, S. Ohlmacher, and I. Saporta-Eksten. 2017. "Business-Level Expectations and Uncertainty." .

Bloom, Nicholas. 2001. "The Real Options Effect of Uncertainty on Investment and Labor Demand." *SSRN* .

———. 2009. "The Impact of Uncertainty Shocks." *Econometrica* 77:623–685.

———. 2014. "Fluctuations in Uncertainty." *The Journal of Economic Perspectives* 28 (2):153–175.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry. 2018. "Really Uncertain Business Cycles." *Econometrica* 86 (3):1031–1065.

Bond, S., R. Moessner, H. Mumtaz, and M. Syed. 2005. "Microeconometric Evidence on Uncertainty and Investment." *Institute for Fiscal Studies, London, UK* .

Born, B. and J. Pfeifer. 2014. "Policy Risk and the Business Cycle." *Journal of Monetary Economics* 68:68–85.

Buera, F. J., J. P. Kaboski, and Y. Shin. 2011. "Finance and Development: A Tale of Two Sectors." *American Economic Review* 101 (5):1964–2002.

Buera, F. J. and B. Moll. 2015. "Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity." *American Economic Journal: Macroeconomics* 7 (3):1–42.

Caballero, R. J. and E. M.R.A Engel. 1999. "Explaining Investment Dynamics in US Manufacturing: a Generalized (S, s) Approach." *Econometrica* 67 (4):783–826.

Caballero, Ricardo J. 1991. "On the Sign of the Investment-Uncertainty Relationship." *The American Economic Review* 81 (1):279–288.

Caplin, A. and J. Leahy. 1993. "Sectoral Shocks, Learning, and Aggregate Fluctuations." *Review of Economic Studies* 60 (4):777–794.

Carvalho, V. M. and B. Grassi. 2017. "Large Firm Dynamics and the Business Cycle." .

Chen, C., T. Senga, C. Sun, and H. Zhang. 2018. "Uncertainty, Imperfect Information, and Learning in the International Market." .

Cooper, R. W. and J. C. Haltiwanger. 2006. "On the Nature of Capital Adjustment Costs." *Review of Economic Studies* 73 (3):611–633.

Cordoba, J. C. 2008. "A Generalized Gibrat's Law." *International Economic Review* 49 (4):1463–1468.

David, Joel M., Hugo A. Hopenhayn, and Venky Venkateswaran. 2016. "Information, Misallocation, and Aggregate Productivity." *The Quarterly Journal of Economics* 131 (2):943–1006.

David, Joel M. and Venky Venkateswaran. 2019. "The sources of capital misallocation." *American Economic Review* 109 (7):2531–2567.

Dixit, Avinash K. and Robert S. Pindyck. 1994. *Investment under Uncertainty.* Princeton University Press.

Fajgelbaum, P., E. Schaal, and M. Taschereau-Dumouchel. 2017. "Uncertainty Traps." *Quarterly Journal of Economics* 132 (4):1641–1692.

Fernandez-Villaverde, J., P. Guerron-Quintana, J. F. Rubio-Ramirez, and M. Uribe. 2011. "Risk

Matters: The Real Effects of Volatility Shocks." *American Economic Review* 101 (6):2530–2561.

Gilchrist, S. and J. C. Williams. 2000. "Putty-clay and investment: A business cycle analysis." *Journal of Political Economy* 108 (5):928–960.

Gilchrist, Simon and John C. Williams. 2005. "Investment, capacity, and uncertainty: a putty–clay approach." *Review of Economic Dynamics* 8 (1):1–27.

Giordani, P. and P. Soderlind. 2003. "Inflation Forecast Uncertainty." *European Economic Review* 47 (6):1037–1059.

Gomme, P., B. Ravikumar, and P. Rupert. 2011. "The Return to Capital and the Business Cycle." *Review of Economic Dynamics* 14 (2):262–278.

Gourio. 2008. "Estimating Firm-Level Risk." *Boston University mimeo* .

Grossman, S. J. and J. E. Stiglitz. 1980. "On the Impossibility of Informationally Efficient Markets." *American Economic Review* 70 (3):393–408.

Guiso, L. and G. Parigi. 1999. "Investment and Demand Uncertainty." *Quarterly Journal of Economics* 114 (1):185–227.

Hansen, G. D. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics* 16 (3):309–327.

Hartman, Richard. 1972. "The effects of price and cost uncertainty on investment." *Journal of Economic Theory* 5 (2):258–266.

———. 1978. "Investment Neutrality of Business Income Taxes." *The Quarterly journal of economics* 92 (2):245–260.

Hayashi, Fumio. 1982. "Tobin's Marginal q and Average q: A Neoclassical Interpretation." *Econometrica* .

Hellwig, Christian, Sebastian Kohls, and Laura Veldkamp. 2012. "Information Choice Technologies." *American Economic Review* 102 (3):35–40.

Hopenhayn, H. A. 1992. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium." *Econometrica* 60 (5):1127–1150.

Houthakker, H.S. 1955. "The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis." *The Review of Economic Studies* 23 (1):27–31.

Hsieh, C. and P. J. Klenow. 2009. "Misallocation and Manufacturing TFP in China and India." *Quarterly Journal of Economics* 124 (4):1403–1448.

Ilut, C., M. Kehrig, and M. Schneider. 2017. "Slow to Hire, Quick to Fire: Employment Dynamics with Asymmetric Responses to News." *Journal of Political Economy* forthcoming.

Ilut, C. and H. Saijo. 2018. "Learning, Confidence, and Business Cycles." .

Janunts, M. 2010. *Differences of Opinion and Stock Returns*. dissertation, Université de Neuchâtel.

Johnson, T. C. 2004. "Forecast Dispersion and the Cross Section of Expected Returns." *Journal of Finance* 59 (5):1957–1978.

Jovanovic, B. 1982. "Selection and the Evolution of Industry." *Econometrica* 50 (3):649–670.

Kacperczyk, Marcin, Stijn Van Nieuwerburgh, and Laura Veldkamp. 2016. "A Rational Theory of Mutual Funds' Attention Allocation." *Econometrica* 84 (2):571–626.

Kehrig, M. 2015. "The Cyclical Nature of the Productivity Distribution." .

Kellogg, R. 2014. "The Effect of Uncertainty on Investment: Evidence from Texas Oil Drilling." *American Economic Review* 104 (6):1698–1734.

Keynes, J. M. 1936. "The General Theory of Employment, Interest, and Money." .

Khan, A., T. Senga, and J. K. Thomas. 2016. "Default Risk and Aggregate Fluctuations in an Economy with Production Heterogeneity." .

Khan, A. and J. K. Thomas. 2013. "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity." *Journal of Political Economy* 121 (6):1055–1107.

Khan, Aubhik and Julia K. Thomas. 2008. "Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics." *Econometrica* 76 (2):395–436.

Kozlowski, J., L. Veldkamp, and V. Venkateswaran. 2018. "The Tail that Wags the Economy: Belief-Driven Business Cycles and Persistent Stagnation." .

Krusell, P. and A. A. Smith. 1997. "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns." *Macroeconomic Dynamics* 1 (02):387–422.

———. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy* 106 (5):867–896.

Leahy, John V and Toni M Whited. 1996. "The effect of uncertainty on investment: Some stylized facts." *Journal of Money, Credit, and Banking* 28 (1):64–83.

Lee, Jaewoo and Kwanho Shin. 2000. "The Role of a Variable Input in the Relationship between Investment and Uncertainty." *The American Economic Review* 90 (3):667–680.

Li, W. and J. Weinberg. 2003. "Firm-Specific Learning and the Investment Behavior of Large and Small Firms." *International Economic Review* 44 (2):599–625.

Masayuki, Morikawa. 2013. "What type of policy uncertainty matters for business?" .

McFadden, Daniel. 1978. *Cost, Revenue, and Profit Functions.*

Oi, W. Y. 1961. "The Desirability of Price Instability Under Perfect Competition." *Econometrica* 29 (1):58–64.

Olley, G. Steven and Ariel Pakes. 1996. "The Dynamics of Productivity in the Telecommunications Equipment Industry." *Econometrica* 64 (6):1263–1297.

Orlik, A. and L. Veldkamp. 2015. "Understanding Uncertainty Shocks and the Role of Black Swans." .

Paddock, J. L., D. R. Siegel, and J. L. Smith. 1988. "Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases." *Quarterly Journal of Economics* 103 (3):479–508.

Pindyck, Robert S. 1988. "Irreversible Investment, Capacity Choice, and the Value of the Firm." *The American Economic Review* 78 (5):969–985.

———. 1993. "A Note on Competitive Investment under Uncertainty." *The American Economic Review* .

Restuccia, D. and R. Rogerson. 2008. "Policy Distortions and Aggregate Productivity with

Heterogeneous Establishments." *Review of Economic Dynamics* 11 (4):707–720.

Rich, R. and J. Tracy. 2010. "The Relationships among Expected Inflation, Disagreement, and Uncertainty: Evidence from Matched Point and Density Forecasts." *Review of Economics and Statistics* 92 (1):200–207.

Rogerson, R. 1988. "Indivisible Labor, Lotteries and Equilibrium." *Journal of Monetary Economics* 21 (1):3–16.

Saijo, H. 2017. "The uncertainty Multiplier and Business Cycles." *Journal of Economic Dynamics and Control* 78:1–25.

Schaal, E. 2017. "Uncertainty and Unemployment." *Econometrica* 85 (6):1675–1721.

Schelkle, Thomas. 2017. "Measuring Factor Misallocation across Industries: General Methods and Evidence on the Great Recession." .

Senga, Tatsuro. 2018. "A New Look at Uncertainty Shocks: Imperfect Information and Misallocation." *Queen Mary University of London mimeo* .

Stein, L. C.D. and E. C. Stone. 2013. "The Effect of Uncertainty on Investment, Hiring, and R&D: Causal Evidence from Equity Options." .

Terry, S. J. 2017. "Alternative Methods for Solving Heterogeneous Firm Models." *Journal of Money, Credit and Banking* 49 (6):1081–1111.

Timoshenko, O. A. 2015. "Product Switching in a Model of Learning." *Journal of International Economics* 95 (2):233–249.

Van Nieuwerburgh, S. and L. Veldkamp. 2006. "Learning Asymmetries in Real Business Cycles." *Journal of Monetary Economics* 53 (4):753–772.

Vavra, Joseph. 2014. "Inflation dynamics and time-varying volatility: New evidence and an Ss interpretation." *Quarterly Journal of Economics* 129 (1):215–258.

Zarnowitz, V. and L. A. Lambros. 1987. "Consensus and Uncertainty in Economic Prediction." *Journal of Political Economy* 95 (3):591–621.

Zwick, Eric and James Mahon. 2017. "Tax policy and heterogeneous investment behavior." *American Economic Review* 107 (1):217–248.

Table 2: : Descriptive Statistics

	mean	sd	p5	p25	p50	p75	p95
Sales (mil. $)	2592.27	5890.19	29.31	183.65	612.83	2083.27	12349.06
Total assets (mil. $)	2783.71	6532.04	52.95	195.40	602.89	2119.88	13381.14
Employment (thous.)	9.34	19.24	0.09	0.59	2.31	8.20	45.50
Age	9.26	8.91	1.00	3.00	6.00	13.00	28.00
Years	17.51	12.72	2.00	7.00	14.00	25.00	45.00
Analyst coverage (#)	7.95	6.83	2.00	3.00	5.42	10.58	22.58
Leverage	0.23	0.20	0.00	0.04	0.20	0.35	0.61
ROA	1.61	12.89	-24.50	0.71	3.58	7.20	14.89
Fdis_cv	39.72	65.08	1.74	5.97	14.49	38.23	196.43
Fdis_sd	28.19	46.21	1.25	5.50	13.00	30.33	107.18
FE_pct	53.00	108.07	0.74	4.68	16.86	52.41	225.37
FE_roa	4.07	9.17	0.03	0.23	0.97	3.58	18.52

Note: The table above shows the cross-sectional moments of the firm-by-year panel. The panel data is constructed by merging data from Compustat, CRSP, and I/B/E/S, resulting in an unbalanced panel of $11,938$ firms between 1976 and 2022. *Sales* and *Total assets* are in millions of 2015 dollars, being deflated with GDP deflator (USAGDPDEFAISMEI, taken from FRED)). *Age* is the number of years calculated from the first year of observation. *Years* is the number of years during which observations can be found. *Analyst coverage* is the number of analysts who reported earnings forecasts. *Leverage* is defined as long-term debt plus current liabilities divided by total assets. *ROA* is calculated as earnings (= street earnings per share (EPS) multiplied by the number of outstanding shares) divided by total assets. $Fdis^{CV}$ is the coefficient of variation of earnings (EPS) forecast dispersion across analysts, and $Fdis^{SD}$ is the standard deviation of earnings (EPS) forecasts. FE^{pct} denotes the forecast error on realized earnings, while FE^{roa} is in terms of ROA. All data are winsorized at the 1 percent level.

Table 3: : Subsamples-Descriptive Statistics

	Low		**High**	
	mean	sd	mean	sd
Sales (mil. $)	3988.71	7642.27	1313.45	3734.52
Total assets (mil. $)	4448.92	8474.57	1661.51	3734.67
Employment (thous.)	12.70	22.66	3.75	9.96
Age	14.19	10.42	9.53	8.29
Years	21.44	11.80	15.02	9.64
Analyst coverage (#)	10.84	7.78	6.57	5.66
Leverage	0.21	0.19	0.22	0.23

Note: *Sales* and *Total assets* are in millions of 2015 dollars, being deflated with GDP deflator (USAGDPDEFAISMEI, taken from FRED)). *Age* is the number of years calculated from the first year of observation. *Years* is the number of years during which observations can be found. *Analyst coverage* is the number of analysts who reported earnings forecasts. *Leverage* is defined as long-term debt plus current liabilities divided by total assets.

Table 4: : Uncertainty fluctuates over time in a countercyclical fashion

	Mean		**S.D.**		**Skewness**	
	(1)	(2)	(3)	(4)	(5)	(6)
	Forecast dispersion	**Forecast error**	**Forecast dispersion**	**Forecast error**	**Forecast dispersion**	**Forecast error**
***GDP* growth**	-0.494***	-0.342**	-0.430***	-0.061	0.344**	0.034
	(0.291)	(0.0336)	(0.293)	(0.0566)	(2.046)	(5.309)
Observations	46	46	46	46	46	46
R^2	.244	.117	.185	3.8e-03	.118	1.1e-03

	Mean		**S.D.**		**Skewness**	
	(1)	(2)	(3)	(4)	(5)	(6)
	Forecast dispersion	**Forecast error**	**Forecast dispersion**	**Forecast error**	**Forecast dispersion**	**Forecast error**
log(GDP)	-0.562***	-0.074	-0.612***	0.052	0.538***	-0.105
	(0.287)	(0.0370)	(0.266)	(0.0587)	(1.903)	(5.471)
Observations	46	46	46	46	46	46
R^2	.316	5.5e-03	.375	2.7e-03	.289	.011

Note: Dependent variables are the cross-sectional mean, standard deviation (SD), and skewness of the forecast dispersion measure ($\mathbf{Fdis^{cv}}$) and forecast error measure ($\mathbf{FE^{roa}}$). They are regressed on GDP growth is calculated as the growth rate of real gross domestic product (GDPCA taken from FRED) and log(GDP) is the HP-filtered series of real gross domestic product (GDPCA taken from FRED). Standard errors are given in parentheses.

Table 5: Common Parameters

Parameter	Description	Target	Parameter Value
β	Risk free real rate	Real interest rate: 0.04	0.96
α	Isoelastic demand	Isoelastic demand with markup: 0.33	0.25
ν	Capital (labour) share	Capital share: 1/3 (Labour share: 2/3)	0.50
δ	Depreciation rate	Annual depreciation: 0.10	0.10
η	Hours worked	Hours worked: 1/3	2.00

Table 6: Micro Parameters

Parameter	Description	Source	Value
π	Yearly persistence of firm-level productivity	Compustat-IBES	0.10
σ_ε	Cross-sectional dispersion of firm-level productivity	Compustat-IBES	0.45
σ_L	Cross-sectional average of $\tilde{v}/\sigma_\varepsilon^2 = 0.41$	Compustat-IBES	0.28
σ_H	$\sigma_H/\sigma_L = 2.1$	Compustat-IBES	0.59
ρ_L	Yearly transition probability	Compustat-IBES	0.70
ρ_H	Yearly transition probability	Compustat-IBES	0.70

Table 7: Unconditional Business Cycle Moments

	U.S. Data, 1976 to 2022			**Model Simulation**		
x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$Corr(x,Y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(Y)}$	$Corr(x,Y)$
Y	1.857	1.000	1.000	1.562	1.000	1.000
C	1.868	1.006	0.909	0.898	0.575	0.635
I	7.858	4.231	0.825	4.351	2.786	0.861
N	2.735	1.472	0.852	0.635	0.925	0.819

Note: The above table shows the data and model business cycle moments of output Y, consumption C, investment I, and hours worked N. $\sigma(x)$ is the standard deviation of x, and $\sigma(x)/\sigma(Y)$ is the relative standard deviation to that of Y, and $Corr(x,Y)$ is the contemporaneous correlation of x with Y. The model moments are obtained from a 1,000-period unconditional simulation using the solution of the model. All series are HP-filtered in logs with a smoothing parameter of 100. The data used to generate the above moments are: (1) real gross domestic product (GDPCA taken from FRED), (2) real gross private domestic investment (GPDICA taken from FRED), (3) real personal consumption expenditures (PCECCA taken from FRED), and (4) total nonfarm business sector hours (HOANBS taken from FRED but anuualized).

Online Appendix

A Empirics

A.1 Data Sources and Variable Construction

In this paper, I use different data sources to construct the uncertainty measures at the firm-year level. The main data sources are summarized in Table 8: (1) Institutional Brokers Estimate System (I/B/E/S), (2) Compustat, (3) Center for Research in Security Prices (CRSP), and (4) OptionMetrics. In particular, earnings forecasts by analysts and the associated earnings outcomes are mainly from the I/B/E/S. Stock market data and implied volatility data are respectively from the CRSP database and the OptionMetrics. Lastly, firm accounting information is largely taken from the Compustat database. In the following, for each data category, I explain the specific sources and their variables step by step.

Table 8: : Firm-level uncertainty measures

	Description	Data Source
$\mathbf{Fdis^{CV}}$	Coefficient of variation of earnings forecasts across analysts	I/B/E/S
$\mathbf{Fdis^{SD}}$	Standard deviation of earnings forecasts across analysts	I/B/E/S
$\mathbf{FE^{pct}}$	Percentage deviation of realized earnings from consensus forecast	I/B/E/S
$\mathbf{FE^{log}}$	Log deviation of realized earnings from consensus forecast	I/B/E/S
$\mathbf{FE^{roa}}$	Percentage point between realized ROA and consensus ROA forecast	I/B/E/S+Compustat
VOL	Annualized standard deviation of daily stock returns	CRSP
IV	Annual average of 30-day options-implied volatility	OptionMetrics
V	Structurally-estimated error variance using DHV (2016) approach	Compustat

Company Financial Information The firm accounting and stock price data are taken from the CRSP/Compustat Merged database (CCM). Specifically, I use the Fundamentals Annual Database (FAD) and the Security Monthly Database (SMD). From the FAD, I take (1) $SALE$ – net sales, (2) AT – total assets, (3) $PPEGT$ – gross property, plant, and equipment, (4) $CSHO$ – common shares outstanding, (5) EMP – employees, (6) DLC – debt in current liabilities, (7) $DLTT$ – long-term debt. In the structural estimation of uncertainty which will be discussed below, I add the relevant variables from the SMD. In that, I retrieve (1) $AJEXM$ – cumulative adjustment factor, (2) $TRFM$ – total return factor, and (3) $PRCCM$ – closing stock.

To construct the sample of firms, I exclude foreign firms as well as utilities and financial firms. In addition, I only consider firms reporting positive values for $SALE$, AT, $PPEGT$, and

CSHO.[23] Lastly, for the figures plotting sales and total assets in this paper, I deflate them either by a GDP price implicit deflator or CPI, both taken from FRED.[24] The results are robust regardless of this choice.

Earnings Forecast Information I use the I/B/E/S summary file to get the key variables that are required to construct the firm-level uncertainty measures based on earnings forecasting. These variables are, (1) the standard deviation of earnings forecasts across analysts (**stdev**), (2) the median earnings forecast (**medest**), and (3) the realized earnings records (**actual**). Additionally, I use the data on the highest, the lowest, and the mean earnings forecasts (**highest**, **lowest**, **meanest**), as well as on the number of analysts (**numana**). All these data items are in terms of earnings per share (EPS) expressed in dollars, except **numana**.

Below, I summarize how to construct the forecast-based uncertainty measures at the firm level.

1. the standard deviation of earnings forecasts across analysts

$$\mathbf{Fdis^{SD}} = \mathbf{stdev}$$

2. the coefficient of variation of earnings forecasts across analysts

$$\mathbf{Fdis^{CV}} = \frac{\mathbf{Fdis^{SD}}}{|\mathbf{medest}|}$$

3. the percentage point between the realized ROA and the consensus ROA forecast

$$\mathbf{FE^{roa}} = \frac{|(\mathbf{actual} - \mathbf{medest})| * \mathbf{csho}}{\mathbf{at}}$$

4. the log deviation of the realized earnings from the consensus forecast

$$\mathbf{FE^{log}} = |\log\left(\frac{\mathbf{actual}}{\mathbf{medest}}\right)|$$

5. the percentage deviation of the realized earnings from the consensus forecast

$$\mathbf{FE^{pct}} = \left|\frac{\mathbf{actual}}{\mathbf{medest}} - 1\right|$$

Once ($\mathbf{Fdis^{SD}}$, $\mathbf{Fdis^{CV}}$, $\mathbf{FE^{ROA}}$, $\mathbf{FE^{log}}$, $\mathbf{FE^{pct}}$) are obtained for a given firm-year, I link the dataset to other annual accounting and performance data in Compustat. When aligning the

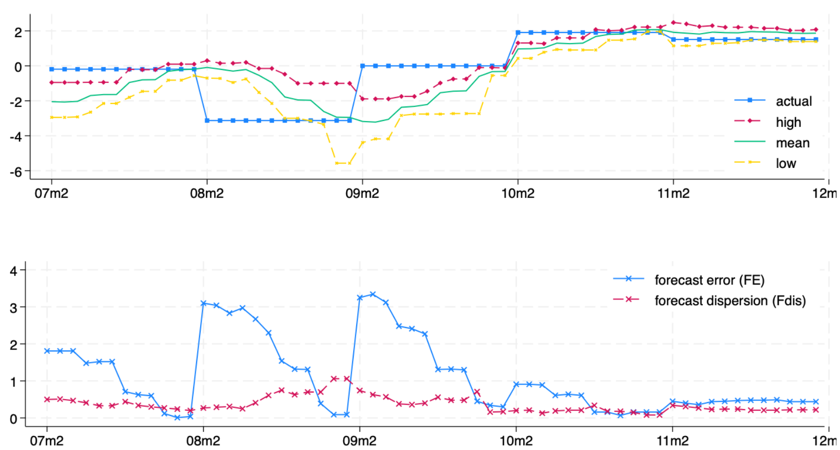
[23] For non-foreign firms, I keep firms only with *fic* or *loc* is "usa" and drop those with *curcd* taking "cad". For non-utilities or financial firms, I drop firms with SIC between 4900 and 4940 or between 6000 and 6999.

[24] In FRED, the corresponding data codes are *USAGDPDEFAISMEI* and *CPALTT01USA661S*.

uncertainty dataset with Compustat, I take the differences in forecasting horizon into account. This is because analysts' forecasts are made throughout each year t and thus there can be different measures of uncertainty by forecast horizon s. For instance, the cross-analyst standard deviation of forecasts is actually denoted as $\textbf{stdev}_{t-s}$, for a given horizon s. In this regard, I need to choose s at which forecasts and the corresponding uncertainty measures are evaluated.

To illustrate the choice of forecast horizon, I begin with an example of a specific firm, Ford Motor Co. (ticker: F), in my panel dataset. Figure 6 reports the analysts' forecasts on Ford Motor Co. in different horizons. Specifically, the top panel of Figure 6 plots ($\textbf{highest}_{t-s}$, $\textbf{meanest}_{t-s}$, $\textbf{lowest}_{t-s}$, $\textbf{actual}_{t-s}$) for $t \in (2007, 2012)$ and $s \in (-1, 10)$. For Ford Motor Co., analysts' forecasts are typically available according to a February-to-January cycle. In other words, the company has its end of fiscal year in December and releases annual earnings records at the end of the following January. After an earnings announcement, analysts start forecasting earnings for the next fiscal year and up until the next earnings announcement after 12 months, while such forecasts are repeatedly updated and revised. Thus, s ranges from -1 to 10 (February to January). In the bottom panel of Figure 6, I plot the corresponding forecast-based measures across different horizons, $\textbf{Fdis}^{\textbf{SD}}{}_{t-s}$ and $\textbf{FE}^{\textbf{ROA}}{}_{t-s}$.

Figure 6: : I/B/E/S data example (Ford Motor Co.)



To consider alternative horizons for measuring forecasts, I examine the patterns of forecasts across different horizons within each year. Figure 7 shows that analysts, on average, start releasing their forecasts within 9 months from each fiscal year-end. In the meantime, forecasts available in 11 and 12 months before the end of each year are relatively scarce. As explained in the case of Ford Motor Co., an earnings announcement ordinarily occurs after the end of each fiscal year and it is typically during the subsequent quarter. Once this earnings season is over, the release of analysts' forecasts follows. Next, it looks less informative to use the forecasts in a 12-month horizon when constructing the associated uncertainty measures. This is because the number of observations dramatically falls after 11-month horizon and the resulting forecast dispersion and forecast error are unusually large. At the 8-month forecast horizon, on the other hand, the pattern seems to indicate that the information related to earnings is revealed and the analysts' forecasts are updated accordingly. That is, we observe a gradual decrease in forecast dispersion and forecast error thereafter.

Based on these observed patterns, I consider the following three alternative forecast horizons. The first one is to use the average forecasts over each firm-year and then to calculate forecasts dispersion and forecast error. While this specification allows me to capture the fluctuations of uncertainty during each year, it also restricts the exact mapping of data into the model that is calibrated at an annual frequency. The second specification is to consider the first month when forecasts are released. For Ford Motor Co., it corresponds to February. This is a simple but transparent method, abstracting from any arbitrary choice of forecast horizon. Again, the mapping into the model may not be perfect because the first month varies across firms in the data. Lastly, as the baseline specification in this paper, I extrapolate and re-scale the first month forecasts to one year-horizon. In this way, the annual model can be better aligned with the empirical uncertainty measures at the firm level. To show the differences between these three alternatives, the forecast errors based on each specification are plotted for Ford Motor Co. in Figure 8. Although each method delivers different levels of forecast error during the sample period, the stylized facts presented in this paper remain robust.

Figure 7: : Pattern of uncertainty by toward

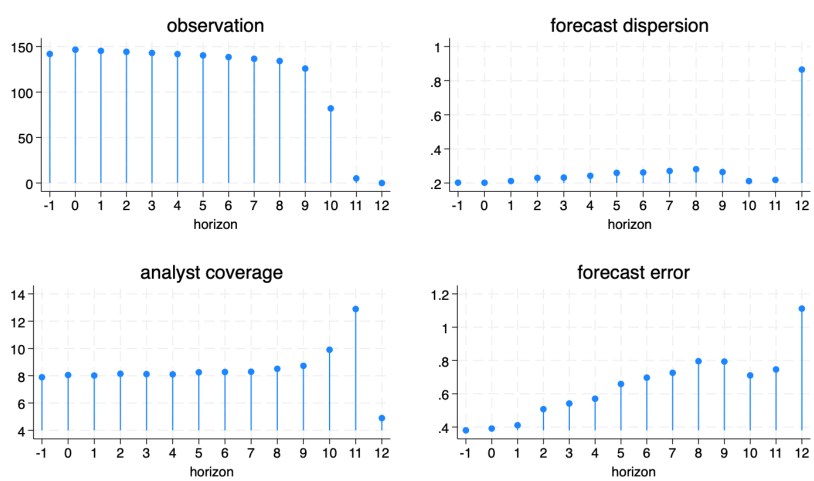
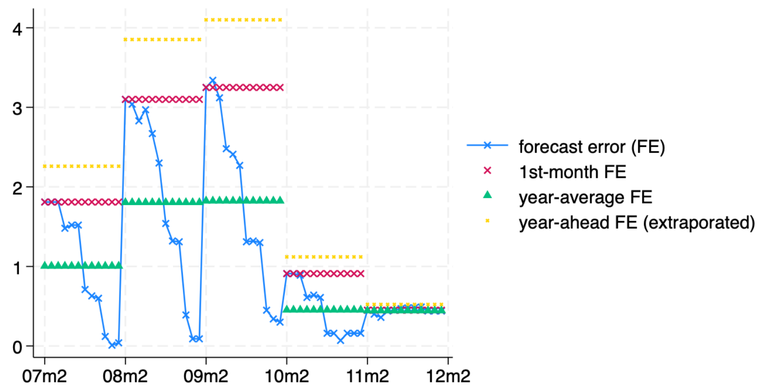


Figure 8: : Ford example - forecast error



Note: The top panel plots the distributions of $fdisp$ across firms in 2007 (dashed line, red), 2008 (solid line, black) and 2009 (solid line, blue). The bottom panel plots the distributions of $ferror$ across firms in 2007 (dashed line, red), 2008 (solid line, black) and 2009 (solid line, blue).

Stock Market Volatility Information For realized stock-return volatility, I use the CRSP Daily Stock database. Stock returns are obtained as the ex-dividend daily returns (**retx**), and I calculate its standard deviation (SD) across all trading days for each firm-year. Specifically, the measure of realized stock-return volatility (**VOL**) is given by,

$$\mathbf{VOL} = \sqrt{250}\ \mathrm{SD}(\mathbf{retx}).$$

To make the unit comparable to that of implied volatility, the above measure is annualized by multiplying $\sqrt{250}$ to the standard deviation of stock returns.

Next, I measure option-implied volatility (**IV**) by using the OptionMetrics database. From the Standardized Options Prices file, a 30-day implied volatility (**impl_volatility**) can be retrieved by specifying (*EXDATE - DATE*) accordingly. I then simply take the yearly average of the implied volatility to get **IV**.

Structural Estimation of Uncertainty Measure As noted earlier, David, Hopenhyan, and Venkateswaran (DHV, 2016) develop a novel identification strategy that uses stock returns to isolate uncertainty from other channels. In particular, DHV extend a model of industry dynamics along the line of Hopenhyan (1992) by introducing information frictions together with stock market. In their model, the stock of a firm is traded among imperfectly informed investors in a noisy way, as in Grossman and Stiglitz (1980) and Albagli, Hellwig, Tsyvinski (2015). Given limited information about their fundamentals, firms make input choices while learning from their stock return movements that aggregate noisy information in financial markets.

The key insight for the structural estimation in DHV (2016) is that a firm's investment decision will be affected by the information contained in stock returns when the firm is more uncertain. Taking this insight into account, I consider a version of the model in DHV, by abstracting from distortions such as fixed adjustment costs and by assuming i.i.d. shocks to fundamentals. This approach allows me to estimate the model-implied uncertainty measure for a given industry-year, whereas DHV focus on comparing a single cross-sectional data across countries. As a result, I have the following equation for the uncertainty measure (V) in the model.

$$\frac{V}{\sigma_\mu^2} = 1 - (\frac{\rho^{pa}}{\rho^{pk}})^2 \tag{14}$$

$\rho^{pa}_{i,t}$ is the correlation between stock returns and changes in fundamentals, $\rho^{pk}_{i,t}$ is the correlation between stock returns and investment, and σ_μ^2 is the variance of i.i.d. innovation of the firm-level fundamentals. The empirical strategy is to retrieve $\rho^{pa}_{i,t}$, $\rho^{pk}_{i,t}$, and σ_μ^2 from my panel data, so that I can isolate V for a given industry-year level. In the following, I describe the data construction required for estimating the uncertainty measure following DHV.

To construct value added (y), fundamental (a), capital stocks (k), investment (i), and stock returns (p), I use data items taken from the merged CRSP/Compustat database (SMD and FAD). As a first step, y is the logarithm of *SALE* multiplied by the intermediate share of $\gamma = 0.5$ as in DHV (2016).

$$y_t = \gamma \log(\text{SALE}_t).$$

For k, the log of capital, I take *PPEGT* in Compustat and then deflate it by using a GDP price implicit deflator in FRED. Given y and k, the fundamental (a) is simply calculated as,

$$a_t = y_t - \alpha k_t.$$

As in DHV, I use $\alpha = 0.83$ for the production parameter value. From *PRCCM*, *TRFM*, and *AJEXM* in Compustat, stock returns (p) are given by,

$$p_t = \log\left(\frac{\text{PRCCM}_t * \text{TRFM}_t}{\text{AJEXM}_t}\right) - \log\left(\frac{\text{PRCCM}_{t-1} * \text{TRFM}_{t-1}}{\text{AJEXM}_{t-1}}\right).$$

Next, I derive the moments in Equation 14, (ρ^{pa}, ρ^{pk}, σ_μ^2), and then retrieve V eventually. To do so, I specify the following autoregressive equations for firm-level stock returns (p), fundamentals (a), and capital (k).

$$p_{j,i,t} = p_{j,i,t-1} + \mu_i + \lambda_t + e^p_{j,i,t}$$

$$a_{j,i,t} = a_{j,i,t-1} + \mu_i + \lambda_t + e^a_{j,i,t}$$

$$k_{j,i,t} = k_{j,i,t-1} + \mu_i + \lambda_t + e^k_{j,i,t}$$

Given the firm fixed effect, μ_i, and the year dummy, λ_t, I recover e^p, e^a, and e^k by estimating the above equations respectively. Then the correlations are formally defined by,

$$\rho^{pa} \equiv \text{Corr}(e^p, e^a)$$

$$\rho^{pk} \equiv \text{Corr}(e^p, e^k).$$

From the cross-sectional dispersion of e^a on an industry-year basis, the standard deviation of the fundamental innovations is,

$$\sigma_\mu \equiv \text{SD}(e^a_{i,t}).$$

Once σ^2_μ, ρ^{pa}, and ρ^{pk} are obtained, it is straightforward to compute the implied V at the industry-year level. The table below summarizes the statistics.

	0						
	mean	sd	p5	p25	p50	p75	p95
variance	0.26	0.13	0.09	0.18	0.25	0.31	0.55
rhopk	0.12	0.43	-0.78	-0.07	0.14	0.34	0.89
rhopa	-0.01	0.44	-0.85	-0.24	-0.02	0.21	0.84
V	0.04	0.05	0.00	0.01	0.03	0.06	0.16

A.2 Disagreement ≈ uncertainty?

Forecast dispersion has nice features in that (1) it is well suited to capture ex-ante uncertainty perceived by agents; and (2) it is available in real time, allowing researcher and policy-makers to monitor the level of uncertainty in a timely manner. However, forecast dispersion reflects market analysts' information set, not business manager's information set. Moreover, forecast dispersion is a measure of disagreement among market analysts. In theory, whether disagreement measures are positively or negatively correlated with uncertainty is ambiguous. It may be the case that all analysts hold very different views about future earnings of a firm while they are very confident about their own view. In such a case, the relationship between forecast dispersion and underlying uncertainty may be negative. Thus, whether we can use dispersion-based measures as a proxy for uncertainty is an empirical question. Answering this question is challenging because we do not directly recognize individual agent's information and thus uncertainty is not easily observable from the data. [25]

[25] To conduct such an empirical exploration, we need data — not only on point forecasts made by analysts but also on the density of each individual analyst's. With such data, we can directly test whether the cross sectional dispersion of point forecasts is indeed correlated with the average of individual analysts' diffusion, say the variance of forecasts at the individual analyst level. While such data are not available for earnings forecasters, to proxy uncertainty about macroeconomic variables, surveys such as the Survey of Professional Forecasters (SPF) have been widely used (see, for example, Zarnowitz and Lambros, 1987; Giordani and Söderlind, 2003; and Rich

To address the above concern, I take the following two approaches to validate the usefulness of forecast dispersion in measuring uncertainty at the firm level. First, I compare my forecast-based measures to other common uncertainty measures in the literature: realized stock market volatility and options-implied volatility. In addition to this reduce-form approach, I also compare my uncertainty measures to that estimated from a theoretical framework of David, Hopenhayn, and Venkateswaran (DHV, 2016). A key identification strategy in DHV is that the investment of an uncertain firm covaries relatively more with the stock prices than its fundamentals. I extend their empirical approach by estimating an uncertainty measure for a given industry-year in my panel dataset. I then compare my forecast dispersion measure to this model-based estimates of uncertainty. This is an important validation as uncertainty is not directly observable in general, which prevents us from solely isolating uncertainty empirically. In this sub-section, I combine the above reduced-form and structural approaches, to examine whether forecast-based uncertainty measures, especially forecast dispersion, effectively capture the existing uncertainty at the firm level.

A.2.1 Reduced-form approach: Stock-price-based uncertainty measures

First, I investigate how forecast-based measures are respectively related to realized stock returns volatility and options-implied volatility, which are common measures of firm-level uncertainty used extensively in previous studies. Concerning realized stock returns volatility, which have been used in Leahy and Whited (1996) and Bloom, Bond and Van Reenen (2007), I take daily stock returns data from the Center for Research in Securities Prices (CRSP) database. I then calculate the realized stock market volatility (= **VOL**) as the annualized standard deviation of daily stock returns of firms. For options-implied volatility, I take daily implied volatility data from OptionMetrics as in Paddock, Siegel and Smith (1988), Bloom (2009), Stein and Stone (2013), and Kellogg (2014). I obtain the options-implied volatility (= **IV**), measured as the average 30-day options-implied volatility of a firm during each year.

By linking these datasets with my panel, I regress the forecast dispersion measure on the realized stock returns volatility and the options-implied volatility. From Table 9, I find that there is a strong positive relationship between the forecast dispersion measure and the realized stock returns volatility and the options-implied volatility at the firm level. This relationship is robust to including year fixed effects, industry fixed effects, firm fixed effects, and controls for firm size and age as in columns (1) to (4) of Table 9. In columns (5) through (8), I instead regress the forecast error measure on the above volatility measures of uncertainty. As in the case of the forecast dispersion measure, I find a strong positive relationship among these variables. All in all,

and Tracy, 2010). For example, Zarnowitz and Lambros (1987) show a positive relationship between forecast dispersion and uncertainty, while Rich and Tracy (2010) find little evidence in support of using disagreement to measure uncertainty. I follow the literature in using earnings forecasts to build a proxy for firm-level uncertainty, as in Johnson (2004), Bond at al. (2005), and Janunts (2010).

from this reduced-form approach, my forecast-based measures of uncertainty, forecast dispersion and forecast error, robustly covary with the existing measures of firm-level uncertainty.[26]

Table 9: : Forecast disagreement and forecast errors covary with realized stock market volatility and options-implied volatility

	Forecast dispersion				Forecast error			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Realized stock market vol.	0.351***		0.297***		0.312***		0.263***	
	(0.0175)		(0.0223)		(0.0149)		(0.0195)	
Options implied vol.		0.369***		0.321***		0.330***		0.270***
		(0.0249)		(0.0343)		(0.0198)		(0.0283)
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Industry FE	Y	Y	N	N	Y	Y	N	N
Firm FE	N	N	Y	Y	N	N	Y	Y
Observations	77514	40353	77514	40353	68782	36586	68782	36586
R^2	0.159	0.177	0.383	0.408	0.125	0.145	0.354	0.371

Note: Forecast dispersion is the coefficient of variation of earnings (EPS) forecast dispersion across analysts, calculated as $\mathbf{Fdis^{cv}}$. Forecast error is the percentage point deviation of the realized ROA from the consensus ROA forecast, calculated as $\mathbf{FE^{roa}}$. Realized stock market vol. is the annualized standard deviation of daily stock returns of firm, defined as **VOL**; and option implied vol. is the average 30-day options-implied volatility of a firm during each year, defined as **IV**.

A.2.2 Structural approach: Model-based uncertainty measure

To further validate whether forecast dispersion is a good proxy for firm-level uncertainty, I take a structural approach based on DHV (2016).[27] Their key insight is that investment decisions covary more strongly with the stock prices than fundamentals when firms are more uncertain. While they compare uncertainty across countries in 2012, I extend their approach by estimating uncertainty for a given industry-year in the US, and compare it to my forecast-based measures of uncertainty. To do so, I take a version of DHV's model wherein firm-level fundamentals is assumed to be i.i.d., while preserving the above insight in the estimation. This approach leads to an analytical expression for measuring uncertainty as below.

$$\frac{V}{\sigma_\mu^2} = 1 - (\frac{\rho^{pa}}{\rho^{pk}})^2 \tag{15}$$

where $\rho_{i,t}^{pa}$ is the correlation between stock returns and changes in fundamentals, $\rho_{i,t}^{pk}$ is the correlation between stock returns and investment, σ_μ^2 is the variance of i.i.d. innovation of the

[26]The results are robust to using the standard deviation instead of the coefficient of variation when I measure forecast dispersion. See Appendix A.2.

[27]DHV (2016) build a model a la Grossman and Stiglitz (1980) and Albagli, Hellwig, Tsyvinski (2015), to estimate the model parameters that determine the level of uncertainty. In particular, they estimate their model to gauge the level of uncertainty in the US and then compare it to those in China and India.

firm-level fundamental, and V is the measure of uncertainty. The empirical strategy is to retrieve $\rho^{pa}_{i,t}$, $\rho^{pk}_{i,t}$, and σ^2_μ from my panel data so that I can isolate V for a given industry-year level.[28]

As in the reduce-form approach, I regress the forecast-based measures on the estimated uncertainty measure in (1), and report the results in Table 10. In columns (1) through (4) of the table, there is a positive relationship between the forecast dispersion measure and the uncertainty measure V, when the latter is structurally estimated from the panel dataset. This result is robust to changing the specifications including year-fixed effects, firm-fixed effects, and firm size and age controls. Moreover, columns (5) to (8) in Table 10 report a positive relationship between the forecast error measure and V. Again, such a relationship is robust to changing the specifications of the regression.[29]

In sum, this subsection aims to establish the usefulness and validation of forecast dispersion as a proxy for firm-level uncertainty. While forecast dispersion may not reflect underlying uncertainty but rather a situation where each analyst is confident but has different views. Nonetheless, the above results, which include both reduced-form and structural approaches; provide support for the view that forecast dispersion is a good measure of firm-level uncertainty. The measures based on forecast dispersion computed from panel data are significantly correlated with commonly-used proxies for firm-level uncertainty including realized stock market volatility and options-implied volatility, and the structurally estimated uncertainty measure using a model with information frictions.

As discussed earlier, forecast dispersion is especially informative and attractive because (1) it is suited for capturing uncertainty perceived by agents when they make decisions and (2) the data is available in real time. These features allow researchers and policymakers to gauge and monitor the level of uncertainty in a timely manner, in contrast to the case for ex-post measures of uncertainty in previous studies. In the following sub-section, I examine the cyclical properties of such uncertainty measures to better understand the nature of uncertainty at the firm-level.

[28] Data constructions and summary statistics for this exercise are included in Appendix A.1.

[29] In DHV(2016)'s model, uncertainty is tightly related to the dispersion of the marginal revenue product of capital. In Appendix A.4, I use my uncertainty measures to test this directly and find that all my measures except **IV** robustly covary with the dispersion of the marginal revenue product of capital. I also show that the dispersion of the marginal revenue product of capital is coutercyclical, which lines up with the recent findings of Schelkle (2017) and Kehrig and Vincent (2018).

Table 10: : Forecast disagreement appears to reflect uncertainty

	Forecast dispersion				Forecast error			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
V	0.049***	0.056***	0.014*	0.014	0.013**	0.033***	0.000	-0.001
	(0.0628)	(0.0628)	(0.0960)	(0.0966)	(0.0465)	(0.0459)	(0.0749)	(0.0746)
Year FE	N	Y	Y	Y	N	Y	Y	Y
Industry FE	N	N	N	N	N	N	N	N
Firm FE	N	N	Y	Y	N	N	Y	Y
Firm Control	N	N	N	Y	N	N	N	Y
Observations	33777	33777	33777	33777	29995	29995	29995	29995
R^2	0.002	0.024	0.439	0.443	0.000	0.021	0.405	0.408

Note: The uncertainty measure V is retlieved from $\frac{V}{\sigma_\mu^2} = 1 - (\frac{\rho^{pa}}{\rho^{pk}})^2$, where where $\rho_{i,t}^{pa}$ is the correlation between stock returns and changes in fundamentals, $\rho_{i,t}^{pk}$ is the correlation between stock returns and investment, σ_μ^2 is the variance of i.i.d. innovation of the firm-level fundamental. See David et al. (2016) for the detail of the model. Data constructions and summary statistics are detailed in Appendix A.1.

A.3 Forecast disagreement is robust when standard deviation is used instead of coefficient of variation

	Forecast dispersion (C.V.)				Forecast dispersion (S.D.)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Realized stock market vol.	0.979***		0.827***		0.491***		0.388***	
	(0.0175)		(0.0223)		(0.0127)		(0.0138)	
Options implied vol.		1.006***		0.875***		0.440***		0.391***
		(0.0249)		(0.0343)		(0.0185)		(0.0219)
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Industry FE	Y	Y	N	N	Y	Y	N	N
Firm FE	N	N	Y	Y	N	N	Y	Y
Observations	77514	40353	77514	40353	77679	40444	77679	40444
R^2	0.159	0.177	0.383	0.408	0.204	0.255	0.569	0.576

Note: Forecast dispersion (C.V.) is the coefficient of variation of earnings (EPS) forecasts across analysts, calculated as $Fdis^{CV}$. Forecast dispersion (S.D.) is the standard deviation of earnings (EPS) forecasts across analysts, calculated as $Fdis^{SD}$. Realized stock market vol. is the annualized standard deviation of daily stock returns of firm, defined as **VOL**; and option implied vol. is the average 30-day options-implied volatility of a firm during each year, defined as **IV**.

A.4 Sub-sample analysis by share price and uncertainty

	Low share price		High share price	
	mean	sd	mean	sd
Sales (mil. $)	1371.67	3775.58	4807.24	8011.02
Total assets (mil. $)	1441.62	4052.43	5262.84	9037.53
Employment (thous.)	5.43	13.11	16.62	25.66
Age	7.43	7.62	12.63	10.05
Years	14.08	10.94	23.83	13.33
Analyst coverage (#)	5.94	5.03	11.65	8.06
Leverage	0.23	0.21	0.22	0.17
ROA	-0.90	14.73	5.65	7.61
Forecast dispersion (C.V.)	51.53	72.72	18.29	40.15
Forecast dispersion (S.D.)	0.30	0.49	0.24	0.41
Forecast error: ROA (pc. deviation)	5.64	10.84	1.22	3.29
Forecast error: EPS (pc. deviation)	0.71	1.25	0.20	0.52
Forecast error: EPS (log deviation)	0.44	0.54	0.18	0.32

Note: The Table above shows the sample mean and the standard deviation of the key variables for the subsamples of firms distinguished by share prices. Specifically, I divide the sample in 2012 by the cross-sectional mean share price and the statistics are separately reported by low and high share price. *Sales* and *Total assets* are in millions of 2010 dollars. *Age* is the number of years calculated from the first year of observation. *Years* is the number of years during which observations can be found. *Analyst coverage* is the number of analysts who reported earnings forecasts. *Leverage* is defined as long-term debt plus current liabilities divided by total assets. *ROA* is calculated as earnings (= street earnings per share (EPS) multiplied by the number of outstanding shares) divided by total assets. Forecast dispersion (C.V.) is the coefficient of variation of earnings (EPS) forecast dispersion across analysts. Forecast dispersion (S.D.) is the standard deviation of earnings (EPS) forecasts. Forecast error: ROA (pc. deviation) is the percentage point between the realized ROA and the consensus ROA forecast. Forecast error: EPS (pc. deviation) denotes the percentage deviation of the realized earnings from the consensus forecast. Forecast error: EPS (log. deviation) denotes the log deviation of the realized earnings from the consensus forecast. All data are winsorized at the 1 percent level.

Table 11: : High-low share price subsamples

	Mean		S.D.		Skewness	
	(1)	(2)	(3)	(4)	(5)	(6)
	Low share price	**High share price**	**Low share price**	**High share price**	**Low share price**	**High share price**
***GDP* growth**	-0.438***	-0.415***	-0.376***	-0.392***	0.360**	0.236
	(40.82)	(30.91)	(33.39)	(54.82)	(2.165)	(8.470)
Observations	46	46	46	46	46	46
R^2	.192	.172	.142	.153	.13	.056

	Mean		S.D.		Skewness	
	(1)	(2)	(3)	(4)	(5)	(6)
	Low share price	**High share price**	**Low share price**	**High share price**	**Low share price**	**High share price**
***GDP* growth**	-0.532***	-0.596***	-0.340**	-0.392***	0.498***	0.392***
	(0.313)	(0.215)	(0.230)	(0.323)	(1.215)	(4.686)
Observations	46	46	46	46	46	46
R^2	.284	.356	.115	.154	.248	.154

Note: Dependent variables are the cross-sectional mean, standard deviation (SD), and skewness of the forecast dispersion measure ($Fdis^{cv}$) and forecast error measure (FE^{roa}). They are regressed, group by group, on GDP growth is calculated as the growth rate of real gross domestic product (GDPCA taken from FRED). Standard errors are given in parentheses.

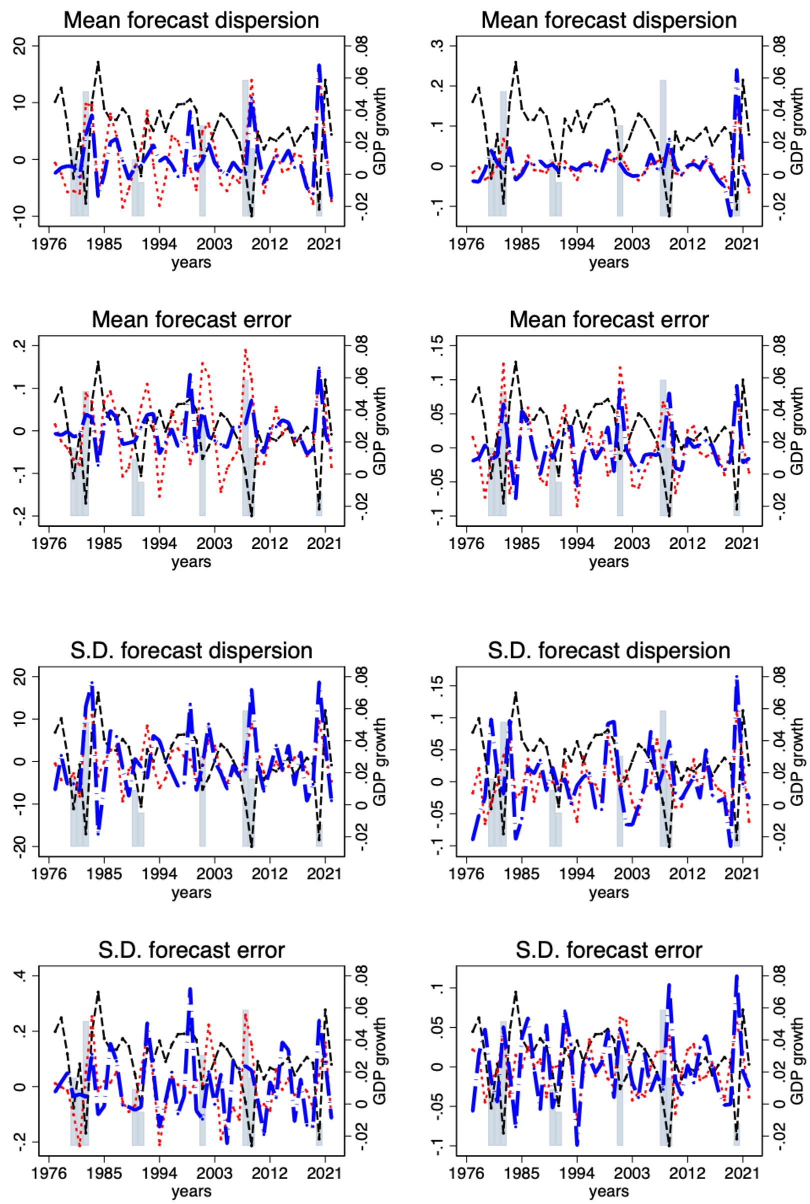
Table 12: : High-low uncertainty subsamples

	Mean		S.D.		Skewness	
	(1)	(2)	(3)	(4)	(5)	(6)
	Low share price	**High share price**	**Low share price**	**High share price**	**Low share price**	**High share price**
***GDP* growth**	-0.541***	-0.406***	-0.477***	-0.285*	0.137	0.386***
	(11.07)	(89.40)	(8.294)	(24.14)	(0.501)	(1.749)
Observations	46	46	46	46	46	46
R^2	.292	.165	.228	.081	.019	.149

	Mean		S.D.		Skewness	
	(1)	(2)	(3)	(4)	(5)	(6)
	Low share price	**High share price**	**Low share price**	**High share price**	**Low share price**	**High share price**
***GDP* growth**	-0.654***	-0.437***	-0.499***	-0.236	0.502***	0.416***
	(0.200)	(0.591)	(0.208)	(0.280)	(2.223)	(1.220)
Observations	46	46	46	46	46	46
R^2	.428	.191	.249	.055	.252	.173

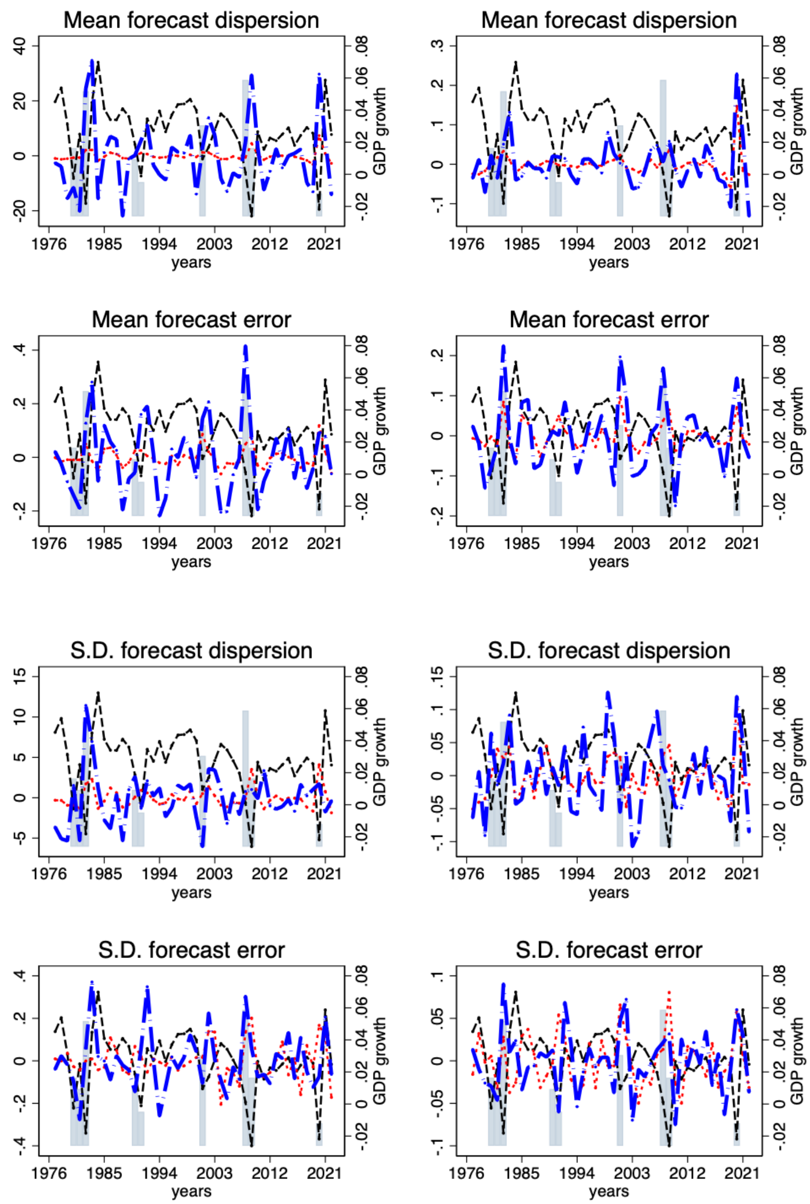
Note: Dependent variables are the cross-sectional mean, standard deviation (SD), and skewness of the forecast dispersion measure ($Fdis^{cv}$) and forecast error measure (FE^{roa}). They are regressed, group by group, on GDP growth is calculated as the growth rate of real gross domestic product (GDPCA taken from FRED). Standard errors are given in parentheses.

Figure 9: : Time-series by share price subsamples



Note: The top (bottom) four panels are the series of the cross-sectional mean (standard deviation). About the top (bottom) fours panel, the top left panel shows the cross-sectional mean (standard deviation) of forecast dispersion $fdis_{cv}$ for firms with high share prices (solid line, red) and for firms with low share prices (dotted line, blue). The top right panel shows the cross-sectional mean (standard deviation) of forecast dispersion $fdis_{sd}$ for firms with high share prices (solid line, red) and for firms with low share prices (dotted line, blue). The bottom left panel shows the cross-sectional mean (standard deviation) of forecast error fe_{roa} for firms with high share prices (solid line, red) and for firms with low share prices (dotted line, blue). The bottom right panel shows the cross-sectional mean (standard deviation) of forecast error fe_{pct} for firms with high share prices (solid line, red) and for firms with low share prices (dotted line, blue). In all panels, HP-filtered real GDP series are plotted (dashed line, black).

Figure 10: : Time-series by uncertainty subsamples



Note: The top (bottom) four panels are the series of the cross-sectional mean (standard deviation). About the top (bottom) fours panel, the top left panel shows the cross-sectional mean (standard deviation) of forecast dispersion $fdis_{cv}$ for firms with high uncertainty (solid line, red) and for firms with low uncertainty (dotted line, blue). The top right panel shows the cross-sectional mean (standard deviation) of forecast dispersion $fdis_{sd}$ for firms with high uncertainty (solid line, red) and for firms with low uncertainty (dotted line, blue). The bottom left panel shows the cross-sectional mean (standard deviation) of forecast error fe_{roa} for firms with high uncertainty (solid line, red) and for firms with low uncertainty (dotted line, blue). The bottom right panel shows the cross-sectional mean (standard deviation) of forecast error fe_{pct} for firms with high uncertainty (solid line, red) and for firms with low uncertainty (dotted line, blue). In all panels, HP-filtered real GDP series are plotted (dashed line, black).

A.5 The marginal revenue products of capital covaies with uncertainty at the industry level

	Forecast dispersion	Forecast error	V	Vol	IV
	(1)	(2)	(3)	(4)	(5)
SD(MPK)	0.051***	0.100***	0.095***	0.063***	0.009
	(0.0176)	(0.00177)	(0.00263)	(0.00491)	(0.00877)
Year FE	Y	Y	Y	Y	Y
Industry FE	Y	Y	Y	Y	Y
Firm FE					
Observations	7862	7861	4385	7859	4645
R^2	0.304	0.265	0.349	0.652	0.657

Note: The above table reports an industry-by-year panel regression. The dependent variable in each column are: (1) the coefficient of variation of earnings (EPS) forecasts across analysts, calculated as $\mathbf{Fdis^{CV}}$; (2) the standard deviation of earnings (EPS) forecasts across analysts, calculated as $\mathbf{Fdis^{SD}}$; (3) the structurally-estimated error variance using DHV (2016) approach **V**; (4) the realized stock market volatility, measured as the annualized standard deviation of daily stock returns of firm, defined as **VOL**; and (5) the option implied volatility, measured as the average 30-day options-implied volatility of a firm during each year, defined as **IV**. They are regressed on the cross-sectional standard deviation of $\log(y/k)$ for a given industry-year, with year fixed effects and industry fixed effects.

A.6 Cyclicality of the marginal revenue products of capital

	Mean			S.D.			Skewness		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Recession	-0.214			-0.413***			-0.007		
	(0.00233)			(0.000894)			(0.00270)		
log(GDP)		0.387***			-0.035			-0.213	
		(0.346)			(0.155)			(0.416)	
***GDP* growth**			0.385***			0.429***			-0.151
			(0.338)			(0.136)			(0.410)
Observations	45	45	45	45	45	45	45	45	45
R^2	.046	.15	.148	.17	1.2e-03	.184	5.4e-05	.046	.023

Note: The above table regress the cross-sectional mean, standard deviation, and skewness measure of $\log(y/k)$ for a given year on the share of quarters within a year (=recession), HP-filtered log GDP series, and GDP growth rate. The underlying samples come from the panel dataset constructed in the main text.

A.7 Sales growth rates are negatively correlated with uncertainty

As shown in Column (1), sales growth rate is negatively associated with the coefficient of variation of analysts forecasts, even after controlling for firm size and age, year fixed effects, and firm fixed effects. The table also shows that the negative relationship is pervasive when other forecast-based measures of uncertainty are considered.

	$\text{Fdis}^{C}V$	$\text{Fdis}^{S}D$	FE_roa	FE_pct
	(1)	(2)	(3)	(4)
uncertainty measure	-0.060***	-0.046***	-1.101***	-0.053***
	(0.00312)	(0.00829)	(0.0516)	(0.00235)
Firm size	-0.198***	-0.195***	-0.207***	-0.197***
	(0.00393)	(0.00388)	(0.00379)	(0.00395)
Firm age	-0.148***	-0.157***	-0.131***	-0.145***
	(0.000924)	(0.000899)	(0.000868)	(0.000880)
Year FE	Y	Y	Y	Y
Firm FE	Y	Y	Y	Y
Observations	67183	67287	66578	66366
R^2	.291595	.2957627	.3060571	.3015976

A.8 Predicting future economic activity

To add a further validation of the uncertainty measures constructed in this paper, I explore how the uncertainty measures based on earnings forecasts predict future economic activity, and compare their performance to other common measures of uncertainty. This involves estimating the following regression equation.

$$y_{t+h} = \alpha + \beta_1 \sigma_t + \sum_{i=1}^{h+1} \gamma_i y_{t-i} + \varepsilon_{t+h} \tag{16}$$

y_{t+h} denotes economic activity with h forecast horizon from year t, σ_t is one of the uncertainty measures that I consider, and ε_{t+h} is the error term. For the uncertainty measures, I include one additional measure of firm-level uncertainty, the conditional heteroskedasticity of firm-level sales which is estimated from a GARCH(1, 1) process in my panel data. Lastly, the model-based measure of uncertainty in Section 1.3.2 is also considered.

According to Table 13, when uncertainty increases as represented by these measures, HP-filtered GDP will slow-down. If we instead use GDP growth for y, the results are similar and robust (Table in the appendix). The significance is weaker in predicting economic activity at the two-year horizon relative to the one-year horizon, and the predictive power of the assumed GARCH process on firm-level sales is less robust when compared to other uncertainty measures. Notice also that the uncertainty measure based on forecast errors is not available in real time as it has to wait until the earnings outcome is revealed, while the forecast dispersion measure is available in real time.

Table 13: : Predicting future economic activity with uncertainty measures

	Current year						Next year					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Fdis_CV	-0.819***						-0.150					
	(-5.87)						(-0.61)					
FE_pct		-0.586***						-0.411*				
		(-3.54)						(-1.89)				
Vol			-0.362*						-0.443*			
			(-1.86)						(-2.00)			
IV				-0.378**						-0.419*		
				(-2.12)						(-1.98)		
Garch					-0.189						0.329*	
					(-1.09)						(1.73)	
VIX						0.347**						0.344*
						(2.09)						(1.80)
Observations	27	27	27	27	27	27	26	26	26	26	26	26
R^2	.711	.534	.373	.397	.314	.394	.11	.221	.233	.232	.203	.211

B Analytical Results

B.1 Proofs

Proof of Lemma 1. If x and y have a bivariate normal distribution

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim \mathbb{N}\left(\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}\right),$$

the conditional distribution of x given y is normal with mean and variance as follows:

$$x \mid y \quad \sim \quad \mathbb{N}(\bar{x} + \frac{\sigma_{xy}}{\sigma_y^2}(y - \bar{y}), \sigma_x^2 - \frac{(\sigma_{xy})^2}{\sigma_y^2}).$$

Since $x = \varepsilon$, $y = \varepsilon + a$, $\bar{x} = \bar{y} = \bar{\varepsilon}$, $\sigma_x^2 = \sigma_\varepsilon^2$, $\sigma_y^2 = \sigma_\varepsilon^2 + \sigma^2$, and $\sigma_{xy} = Cov(\varepsilon, s) = Cov(\varepsilon, \varepsilon + a) = Var(\varepsilon) + Cov(\varepsilon, a) = \sigma_\varepsilon^2$, the firm's conditional expectation can be written as

$$\tilde{\varepsilon}_{jt} \mid s_{jt} \quad \sim \quad \mathbb{N}(\bar{\varepsilon} + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma^2}(s_{jt} - \bar{\varepsilon}), \sigma_\varepsilon^2 - \frac{\sigma_\varepsilon^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma^2}).$$

Simplifying the expression further leads to the conditional mean and variance as follows.

$$\begin{aligned} \bar{\varepsilon} + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma^2}(s_{jt} - \bar{\varepsilon}) &= \big(1 - \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma^2}\big)\bar{\varepsilon} + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma^2} s_{jt} = \frac{\sigma^2}{\sigma_\varepsilon^2 + \sigma^2}\bar{\varepsilon} + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma^2} s_{jt} \\ &= \frac{\tilde{v}}{\sigma_\varepsilon^2}\bar{\varepsilon} + \frac{\tilde{v}}{\sigma^2} s_{jt}, \text{ where } \tilde{v} = \frac{\sigma_\varepsilon^2 \sigma^2}{\sigma_\varepsilon^2 + \sigma^2}. \end{aligned}$$

and

$$\sigma_\varepsilon^2 - \frac{\sigma_\varepsilon^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma^2} = \sigma_\varepsilon^2(1 - \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma^2}) = \sigma_\varepsilon^2 \frac{\sigma^2}{\sigma_\varepsilon^2 + \sigma^2} = \tilde{v}.$$

This proves the lemma in the main text. ■

Proof of Lemma 2.

$$\begin{aligned} Var(\tilde{\varepsilon}_{jt}) &= (\frac{\tilde{v}}{\sigma^2})^2 Var(s_{jt}) = \Big(\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma^2}\Big)^2 (\sigma_\varepsilon^2 + \sigma^2) = \frac{\sigma_\varepsilon^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma^2} \\ &= \frac{\sigma_\varepsilon^2 \sigma_\varepsilon^2 + \sigma_\varepsilon^2 \sigma^2}{\sigma_\varepsilon^2 + \sigma^2} - \frac{\sigma_\varepsilon^2 \sigma^2}{\sigma_\varepsilon^2 + \sigma^2} = \sigma_\varepsilon^2 - \tilde{v}. \end{aligned}$$

To examine how $Var(\tilde{\varepsilon}_{jt})$ varies with σ^2:

$$\begin{aligned}\frac{d}{d\sigma^2}Var(\tilde{\varepsilon}_{jt}) &= \frac{d}{d\sigma^2}(\sigma_\varepsilon^2 - \tilde{v}) = -\frac{d\tilde{v}}{d\sigma^2} \\ &= -\frac{d}{d\sigma^2}\left(\frac{\sigma_\varepsilon^2 \cdot \sigma^2}{\sigma_\varepsilon^2 + \sigma^2}\right) = -\frac{\sigma_\varepsilon^4}{(\sigma_\varepsilon^2 + \sigma^2)^2}.\end{aligned}$$

Since $\sigma_\varepsilon^4 > 0$ (assuming $\sigma_\varepsilon^2 > 0$) and $(\sigma_\varepsilon^2 + \sigma^2)^2 > 0$ for all $\sigma^2 \geq 0$ and $\sigma_\varepsilon^2 > 0$, we have:

$$\frac{d}{d\sigma^2}Var(\tilde{\varepsilon}_{jt}) = -\frac{\sigma_\varepsilon^4}{(\sigma_\varepsilon^2 + \sigma^2)^2} < 0.$$

Therefore, $Var(\tilde{\varepsilon}_{jt})$ is strictly monotonically decreasing in σ^2.

Similarly, to examine how $Var(\tilde{\varepsilon}_{jt})$ varies with σ_ε^2:

$$\begin{aligned}\frac{d}{d\sigma_\varepsilon^2}Var(\tilde{\varepsilon}_{jt}) &= \frac{d}{d\sigma_\varepsilon^2}(\sigma_\varepsilon^2 - \tilde{v}) = 1 - \frac{d\tilde{v}}{d\sigma_\varepsilon^2} \\ &= 1 - \frac{d}{d\sigma_\varepsilon^2}\left(\frac{\sigma_\varepsilon^2 \cdot \sigma^2}{\sigma_\varepsilon^2 + \sigma^2}\right) = 1 - \frac{\sigma^4}{(\sigma_\varepsilon^2 + \sigma^2)^2}.\end{aligned}$$

To determine the sign, note that:

$$1 - \frac{\sigma^4}{(\sigma_\varepsilon^2 + \sigma^2)^2} = \frac{(\sigma_\varepsilon^2 + \sigma^2)^2 - \sigma^4}{(\sigma_\varepsilon^2 + \sigma^2)^2} = \frac{\sigma_\varepsilon^4 + 2\sigma_\varepsilon^2\sigma^2}{(\sigma_\varepsilon^2 + \sigma^2)^2}.$$

Since the numerator $\sigma_\varepsilon^4 + 2\sigma_\varepsilon^2\sigma^2 > 0$ (assuming $\sigma_\varepsilon^2 > 0$) and the denominator $(\sigma_\varepsilon^2 + \sigma^2)^2 > 0$, we conclude:

$$\frac{d}{d\sigma_\varepsilon^2}Var(\tilde{\varepsilon}_{jt}) > 0.$$

Therefore, $Var(\tilde{\varepsilon}_{jt})$ is strictly monotonically increasing in σ_ε^2. ■

Proof of Proposition 1. The firm's objective at time t is to choose labor $\{n_{jt+s}\}_{s=0}^{\infty}$ and investment $\{i_{jt+s}\}_{s=0}^{\infty}$ to maximize the present value of cash flow. Letting r be the interest rate faced by the firm and assumed to be constant over time.

$$\max_{\{n_{jt+s}, i_{jt+s}\}_{s=0}^{\infty}} \quad E_t \sum_{s=0}^{\infty} \frac{1}{(1+r)^s}(z_{jt+s}k_{jt+s}^{\alpha}n_{jt+s}^{\nu} - \omega_{t+s}n_{jt+s} - i_{jt+s}) \tag{17a}$$

$$\text{s.t.} \quad k_{jt+s+1} = (1-\delta)k_{jt+s} + i_{jt+s}, \tag{17b}$$

given an initial condition for k_{jt}.

Taking the first order condition with respect to n_{jt} yields $\nu z_{jt}k_{jt}^{\alpha}n_{jt}^{\nu-1} = \omega_t$. This can be combined with (17a) to rewrite the optimization problem as

$$\max_{\{i_{jt+s}\}_{s=0}^{\infty}} \quad E_t \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} \left[(1-\nu)\left(\frac{\nu}{\omega}\right)^{\frac{\nu}{1-\nu}} z_{jt+s}^{\frac{1}{1-\nu}} k_{jt+s}^{\frac{\alpha}{1-\nu}} - i_{jt+s}\right] \tag{18a}$$

$$\text{s.t.} \qquad k_{jt+s+1} = (1-\delta)k_{jt+s} + i_{jt+s}, \tag{18b}$$

given an initial condition for k_{jt}.

The first order condition is

$$\alpha\left(\frac{\nu}{\omega}\right)^{\frac{\nu}{1-\nu}} E[z_{jt+1}^{\frac{1}{1-\nu}} \mid \tilde{\varepsilon}_{jt}, \tilde{v}] k_{jt+1}^{\frac{\alpha-1+\nu}{1-\nu}} = r + \delta. \tag{19}$$

Dropping firm subscripts for notational simplicity, the relationships derived apply to all firms. Rearranging equation (19) and taking log leads to

$$\begin{aligned} \log k_{t+1} = {} & \frac{\nu}{1-(\alpha+\nu)} \log\left(\frac{\nu}{\omega}\right) + \frac{1-\nu}{1-(\alpha+\nu)} \log E[z_{t+1}^{\frac{1}{1-\nu}} \mid \tilde{\varepsilon}, \tilde{v}] \\ & - \frac{1-\nu}{1-(\alpha+\nu)} \log \frac{(r+\delta)}{\alpha}. \end{aligned} \tag{20}$$

Recall $z = e^{\varepsilon}$ is log normally distributed, as ε has the normal distribution with mean $\bar{\varepsilon}$ and variance σ_{ε}^2. Since the conditional expectation of $z_{t+1}^{\frac{1}{1-\nu}}$ can be written as

$$E[z_{t+1}^{\frac{1}{1-\nu}} \mid \tilde{\varepsilon}, \tilde{v}] = \exp\left(\frac{1}{1-\nu}\tilde{\varepsilon} + \frac{1}{2}\left(\frac{1}{1-\nu}\right)^2 \tilde{v}\right),$$

the second term in equation (20) is

$$\log E[z_{t+1}^{\frac{1}{1-\nu}} \mid \tilde{\varepsilon}, \tilde{v}] = \frac{1}{1-\nu}\tilde{\varepsilon} + \frac{1}{2}\left(\frac{1}{1-\nu}\right)^2 \tilde{v},$$

which leads to the following capital choice equation:

$$\begin{aligned} \log k_{t+1} = {} & \frac{\nu}{1-(\alpha+\nu)} \log\left(\frac{\nu}{\omega}\right) + \frac{1}{1-(\alpha+\nu)} \left[\tilde{\varepsilon} + \frac{1}{2}\left(\frac{1}{1-\nu}\right)\tilde{v}\right] \\ & - \frac{1-\nu}{1-(\alpha+\nu)} \log \frac{(r+\delta)}{\alpha}. \end{aligned} \tag{21}$$

Therefore, we have the joint distribution as follows:

$$\begin{pmatrix} \log z \\ \log k \end{pmatrix} \sim N\left(\begin{pmatrix} \mathbb{E}(\log z) \\ \mathbb{E}(\log k) \end{pmatrix}, \begin{pmatrix} \text{Var}(\log z) & \text{Cov}(\log z, \log k) \\ \text{Cov}(\log z, \log k) & \text{Var}(\log k) \end{pmatrix}\right)$$

, where the mean and covariance terms can be derived as:

$$
\begin{aligned}
\mathbb{E}[\log z] &= \bar{\varepsilon}, \quad \mathrm{Var}[\log z] = \sigma_\varepsilon^2 \\
\mathbb{E}[\log k] &= \underbrace{\frac{\nu}{1-(\alpha+\nu)} \log\left(\frac{\nu}{\omega}\right) - \frac{1-\nu}{1-(\alpha+\nu)} \log \frac{(r+\delta)}{\alpha}}_{\hat{K}_0} + \frac{1}{1-(\alpha+\nu)}\left(\bar{\varepsilon} + (\frac{1}{1-\nu})\frac{\tilde{v}}{2}\right) \\
\mathrm{Var}[\log k] &= \left(\frac{1}{1-(\alpha+\nu)}\right)^2 \mathrm{Var}(\tilde{\varepsilon}) = \left(\frac{1}{1-(\alpha+\nu)}\right)^2 (\sigma_\varepsilon^2 - \tilde{v}) \\
\mathrm{Cov}(\log z, \log k) &= \mathrm{Cov}\left(\varepsilon, \frac{1}{1-(\alpha+\nu)}\tilde{\varepsilon}\right) = \mathrm{Cov}\left(\varepsilon, \frac{1}{1-(\alpha+\nu)}(\frac{\tilde{v}}{\sigma_\varepsilon^2}\bar{\varepsilon} + \frac{\tilde{v}}{\sigma^2}s)\right) \\
&= \mathrm{Cov}\left(\varepsilon, \frac{1}{1-(\alpha+\nu)}\left(\frac{\tilde{v}}{\sigma_\varepsilon^2}\bar{\varepsilon} + \frac{\tilde{v}}{\sigma^2}(\varepsilon + a)\right)\right) \\
&= \frac{1}{1-(\alpha+\nu)}\frac{\tilde{v}}{\sigma^2}\mathrm{Var}\left(\varepsilon\right) = \frac{1}{1-(\alpha+\nu)}\frac{\tilde{v}}{\sigma^2}\sigma_\varepsilon^2 = \frac{1}{1-(\alpha+\nu)}\frac{\sigma_\varepsilon^2\sigma^2}{\sigma_\varepsilon^2+\sigma^2}\frac{\sigma_\varepsilon^2}{\sigma^2} \\
&= \frac{1}{1-(\alpha+\nu)}\frac{\sigma_\varepsilon^2\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma^2} = \frac{1}{1-(\alpha+\nu)}\left(\sigma_\varepsilon^2 - \frac{\sigma_\varepsilon^2\sigma^2}{\sigma_\varepsilon^2+\sigma^2}\right) \\
&= \frac{1}{1-(\alpha+\nu)}(\sigma_\varepsilon^2 - \tilde{v})
\end{aligned}
$$

Aggregate output Y is computed by integrating over the cross-sectional distribution of firms:

$$
Y = \int z k^\alpha n^\nu \, d\mu(z,k) = \left(\frac{\nu}{\omega}\right)^{\frac{\nu}{1-\nu}} \int z^{\frac{1}{1-\nu}} k^{\frac{\alpha}{1-\nu}} \, d\mu(z,k)
$$

where $\mu(z,k)$ represents the joint distribution of productivity and capital across firms.

Since $(\log z, \log k)$ are jointly normal, we can compute the aggregate using the property $\mathbb{E}[e^X] = e^{\mathbb{E}[X]+\frac{1}{2}\mathrm{Var}(X)}$ where $X = \frac{1}{1-\nu}\log z + \frac{\alpha}{1-\nu}\log k$. Thus,

$$
\begin{aligned}
\log Y = &\frac{\nu}{1-\nu}\log\left(\frac{\nu}{\omega}\right) + \mathbb{E}\left[\frac{1}{1-\nu}\log z + \frac{\alpha}{1-\nu}\log k\right] + \frac{1}{2}\mathrm{Var}\left[\frac{1}{1-\nu}\log z + \frac{\alpha}{1-\nu}\log k\right] \\
= &\frac{\nu}{1-\nu}\log\left(\frac{\nu}{\omega}\right) + \frac{1}{1-\nu}\mathbb{E}[\log z] + (\frac{1}{1-\nu})^2\frac{1}{2}Var[\log z] \\
+ &\frac{\alpha}{1-\nu}\mathbb{E}[\log k] + (\frac{\alpha}{1-\nu})^2\frac{1}{2}Var[\log k] \\
+ &\frac{1}{1-\nu}\frac{\alpha}{1-\nu}Cov(\log z, \log k).
\end{aligned} \tag{22}
$$

Let $K_0 = \frac{\nu}{1-\nu}\log\left(\frac{\nu}{\omega}\right) + \frac{\alpha}{1-\nu}\hat{K}_0$. Substituting into (22) the mean and covariance matrix:

$$\log Y = K_0 + \frac{1}{1-\nu}\bar{\varepsilon} + \frac{1}{2}\left(\frac{1}{1-\nu}\right)^2 \sigma_\varepsilon^2 \tag{23}$$

$$+ \frac{\alpha}{1-\nu}\left(\frac{1}{1-(\alpha+\nu)}\bar{\varepsilon} + \frac{1}{2(1-\nu)(1-(\alpha+\nu))}\tilde{v}\right) \tag{24}$$

$$+ \frac{1}{2}\left[\left(\frac{\alpha}{1-\nu}\right)^2\left(\frac{1}{1-(\alpha+\nu)}\right)^2 + \frac{2\alpha}{(1-\nu)^2}\frac{1}{1-(\alpha+\nu)}\right](\sigma_\varepsilon^2 - \tilde{v}). \tag{25}$$

Rearranging and grouping terms to match the effects identified in the main text (Equation (6)), and abstracting from the overall constant K_0:

$$\begin{aligned}
\log Y \approx\ & \underbrace{\left(\frac{1}{1-\nu}\bar{\varepsilon} + \frac{1}{2}\left(\frac{1}{1-\nu}\right)^2\sigma_\varepsilon^2\right)}_{\text{“Jensen effect”}} \\
& + \underbrace{\left(\frac{\alpha}{1-(\alpha+\nu)}\left[\frac{1}{1-\nu}\bar{\varepsilon}\right] + \frac{\alpha}{1-(\alpha+\nu)}\left[\left(\frac{1}{1-\nu}\right)^2\frac{\tilde{v}}{2}\right]\right)}_{\text{“Oi-Hartman-Abel effect”}} \\
& + \underbrace{\left(\frac{1}{2}\left(\frac{1}{1-\nu}\right)^2\left[\frac{\alpha}{1-(\alpha+\nu)}\right]^2 + \frac{1}{(1-\nu)^2}\frac{\alpha}{1-(\alpha+\nu)}\right)\sigma_\varepsilon^2}_{\text{“Reallocation effect”}} \\
& - \underbrace{\left(\frac{1}{2}\left(\frac{1}{1-\nu}\right)^2\left[\frac{\alpha}{1-(\alpha+\nu)}\right]^2 + \frac{1}{(1-\nu)^2}\frac{\alpha}{1-(\alpha+\nu)}\right)\tilde{v}}_{\text{“Uncertainty effect”}}
\end{aligned} \tag{26}$$

To analyze the impact of volatility and noise, we differentiate Equation (26) with respect to σ_ε^2 and σ^2. Let us define the following positive coefficients from Equation (26): The coefficient of σ_ε^2 in the Jensen effect:

$$C_J = \frac{1}{2}\left(\frac{1}{1-\nu}\right)^2 > 0$$

The coefficient of $\tilde{v}$ in the Oi-Hartman-Abel effect:

$$C_{OHA} = \frac{\alpha}{1-(\alpha+\nu)}\left(\frac{1}{1-\nu}\right)^2\frac{1}{2} = \frac{\alpha}{2(1-\nu)^2(1-(\alpha+\nu))} > 0$$

And the coefficient for the Reallocation and Uncertainty effects:

$$C_R = \left(\frac{1}{2}\left(\frac{1}{1-\nu}\right)^2\left[\frac{\alpha}{1-(\alpha+\nu)}\right]^2 + \frac{1}{(1-\nu)^2}\frac{\alpha}{1-(\alpha+\nu)}\right) > 0 \tag{27}$$

Note that the terms in Equation (26) that depend only on $\bar{\varepsilon}$ do not affect these derivatives.

Effect of σ_ε^2 (volatility): Differentiating $\log Y$ (from Equation (26)) with respect to σ_ε^2:

$$\frac{\partial \log Y}{\partial \sigma_\varepsilon^2} = C_J + C_{OHA}\frac{\partial \tilde{v}}{\partial \sigma_\varepsilon^2} + C_R\frac{\partial(\sigma_\varepsilon^2 - \tilde{v})}{\partial \sigma_\varepsilon^2}$$

Since $C_J > 0$, $C_{OHA} > 0$, $C_R > 0$, and both derivatives $\frac{\partial \tilde{v}}{\partial \sigma_\varepsilon^2}$ and $\frac{\partial(\sigma_\varepsilon^2 - \tilde{v})}{\partial \sigma_\varepsilon^2}$ are positive from Lemma 2, $\frac{\partial \log Y}{\partial \sigma_\varepsilon^2}$ is positive. Aggregate output increases with volatility σ_ε^2.

Effect of σ^2 (noise): Differentiating $\log Y$ (from Equation (26)) with respect to σ^2:

$$\frac{\partial \log Y}{\partial \sigma^2} = C_{OHA}\frac{\partial \tilde{v}}{\partial \sigma^2} + C_R\frac{\partial(\sigma_\varepsilon^2 - \tilde{v})}{\partial \sigma^2} = C_{OHA}\frac{\partial \tilde{v}}{\partial \sigma^2} - C_R\frac{\partial \tilde{v}}{\partial \sigma^2} = (C_{OHA} - C_R)\frac{\partial \tilde{v}}{\partial \sigma^2}$$

We used $\frac{\partial(\sigma_\varepsilon^2 - \tilde{v})}{\partial \sigma^2} = -\frac{\partial \tilde{v}}{\partial \sigma^2}$ from Lemma 2. We have: $C_{OHA} = \frac{\alpha}{2(1-\nu)^2(1-(\alpha+\nu))}$ and $C_R = \frac{\alpha^2}{2(1-\nu)^2(1-(\alpha+\nu))^2} + \frac{\alpha}{(1-\nu)^2(1-(\alpha+\nu))} = \frac{\alpha^2}{2(1-\nu)^2(1-(\alpha+\nu))^2} + 2 \cdot C_{OHA}$. Since $\alpha > 0$, $1-\nu > 0$, and $1-(\alpha+\nu) > 0$, the term $\frac{\alpha^2}{2(1-\nu)^2(1-(\alpha+\nu))^2}$ is strictly positive. Thus, $C_R > C_{OHA}$, which implies $(C_{OHA} - C_R) < 0$. Given that $\frac{\partial \tilde{v}}{\partial \sigma^2} > 0$ and $(C_{OHA} - C_R) < 0$, $\frac{\partial \log Y}{\partial \sigma^2} < 0$. Aggregate output decreases with noise σ^2.

Therefore, aggregate output increases with σ_ε^2 and decreases with σ^2. ■

C Numerical Method

In this subsection I describe the solution method used to generate the results in the paper. A common numerical method in computing an equilibrium business cycle model with heterogeneous agents under aggregate uncertainty is to approximate the firm distribution with a finite set of moments such as the mean of asset holdings, following Krusell and Smith's (1997, 1998) heterogeneous household model, or of capital stock as in Khan and Thomas' (2008) heterogeneous firm model. Moreover, Khan and Thomas (2013) extend their original forecasting rules to allow credit shocks in that they introduce ζ_1 and ζ_2 as dummy variables: $\zeta_1 = 1$ if the economy was in a credit crisis in the last period (otherwise $\zeta_1 = 0$). Similarly, $\zeta_2 = 1$ if the economy was in a credit crisis in two periods in the past (otherwise $\zeta_2 = 0$). I follow Khan and Thomas (2013)'s forecasting rules for my two-state Markov process: ordinary states σ^L and uncertain state σ^H.

Let $C(\sigma, \mu)$ and $N(\sigma, \mu)$ be the market-clearing consumption and hours worked. It follows that market-clearing conditions are: (1) the real wage equal the household marginal rate of substitution between leisure and consumption: $w(\sigma, \mu) = D_2U(C(\sigma, \mu), 1 - N(\sigma, \mu))/D_1U(C(\sigma, \mu), 1 - N(\sigma, \mu))$; and (2) firms' state-contingent discount factors are consistent with the household marginal rate of substitution between consumption across states: $d(\sigma', \sigma, \mu) = \beta D_1U(C(\sigma', \mu'), 1 - N(\sigma', \mu'))/D_1U(C(\sigma, \mu), 1 - N(\sigma, \mu))$.

I then define $p(\sigma, \mu) = D_1U(C(\sigma, \mu), 1 - N(\sigma, \mu))$; and, write the real wage as, $w(\sigma, \mu) = D_2U(C(\sigma, \mu), 1 - N(\sigma, \mu))/p(\sigma, \mu)$.

This allows me to reformulate the firm's problem in the main text as follows:

$$\begin{aligned} w(\tilde{\varepsilon}, \tilde{v}, s, k; \sigma, \mu) = \max_{n,k'} \Big[& p(\sigma, \mu)\Big(e^{\varepsilon}k^{\alpha}n^{\nu} - \omega(s, \mu)n + (1-\delta)k - k'\Big) \\ & + (1-\pi)\beta\mathbb{E}_{\sigma'|\sigma} w_0(\tilde{\varepsilon}', \tilde{v}', k'; \sigma', \mu') \\ & + \pi\beta\mathbb{E}_{\sigma'|\sigma} w_0(\tilde{\varepsilon}'_0, \tilde{v}'_0, k'; \sigma', \mu')\Big] \end{aligned} \tag{27}$$

$$\text{subject to :} \quad \mu' = \Gamma(s, \mu). \tag{28}$$

$$w_0(\tilde{\varepsilon}, \tilde{v}, k; \sigma, \mu) = \mathbb{E}_{s|(\tilde{\varepsilon}, \tilde{v})}\, w(\tilde{\varepsilon}, \tilde{v}, s, k; \sigma, \mu). \tag{29}$$

My model includes a distribution, which is in general a high-dimensional object, and I approximate it with the first-moment of the distribution of capital m. As noted, I introduce two dummy variables: ψ_1 and ψ_2 where $\psi_1 = 1$ if the economy was in an uncertain state in the previous period and $\psi_1 = 0$ otherwise. Similarly, $\psi_2 = 1$ if the economy was in an uncertain state two periods in the past and $\psi_2 = 0$ otherwise. Therefore, agents in the model perceive

$(\sigma, m, \psi_1, \psi_2)$ as the economy's aggregate state.

The solution method iterates until the following forecasting rules are converged:

$$\log x_{t+1} = \beta_0^i + \beta_1^i \log x_t + \beta_2^i \psi_{1,t} + \beta_3^i \psi_{2,t}$$

where $x \in \{p, m\}$ and i corresponds to the aggregate state either $\sigma = \sigma_L$ or $\sigma = \sigma_H$. First, firms' value functions are solved in an inner loop using existing forecasting rules for p and m. After isolating firms value functions W from the above problem in the inner loop, we then use such W in an outer loop where I simulate the model by finding equilibrium quantities and prices period by period. That is, with the aggregate state at the beginning of each period, $(\sigma, m, \psi_1, \psi_2)$, I use the forecasting rule and firms value functions W to calculate the future values and let firms choose optimal investment and labor decision: k' and n. With such optimal decisions, I calculate the aggregate quantities to check if the market clearing conditions are satisfied, and until they do, I keep this optimization step to find equilibrium quantities and prices in each period. Once they are obtained, I update the distribution of firms for the next period, which gives m, the endogenous aggregate state at the beginning of the next period, along with exogenous state like σ,ψ_1,ψ_2. From the simulation, a time-series of aggregate variables is obtained and used to estimate new forecasting rules. With new forecasting rules estimated, I go back to an innerloop and move onto an outerloop, and I repeat the process until forecasting rules are converged. Every outerloop starts with a distribution of firms derived from the steady state solution of the model. The equilibrium forecasting rules are presented in Table 14.

Table 14: : Conditional forecasting rules

Rule	State	β_0	β_1	β_2	β_3	S.E.	Adj. R^2
Forecasting p	$\sigma = \sigma_L$	0.69690	-0.40362	0.00042	0.00079	0.00015	0.99996
Forecasting p	$\sigma = \sigma_H$	0.70290	-0.41646	0.00036	0.00064	0.00011	0.99935
Forecasting m'	$\sigma = \sigma_L$	0.05910	0.88466	-0.00190	-0.00221	0.00052	0.99623
Forecasting m'	$\sigma = \sigma_H$	0.06523	0.87142	-0.00197	-0.00200	0.00050	0.99724

Recently, the shape of the distribution of micro-level agents lies at the heart of the debate on the aggregate implications, as, for example, demonstrated by Krueger, Mitman, and Perri (2016). They show that as long as the movement of cross-sectional distributions lines up with the movement of the mean of the distribution, then quasi-aggregation works even for economies with highly skewed distributions. As eloquently explained by Krueger et al. (2016), the distribution of micro-level agents matter in Khan and Thomas (2013) and in my model; however, quasi-aggregation works.

D Business Cycle Statistics: Parameter Robustness Analysis

Table 15: Business Cycle Statistics: Varying σ (Risk Parameter)

Variable	**Statistic**	**imp1636** ($\sigma = 0.10$)	**imp0307** ($\sigma = 0.13$)	**imp1259** ($\sigma = 0.16$)
Output (Y)	$\sigma(x)$	0.681	0.559	0.356
	$\sigma(x)/\sigma(Y)$	1.000	1.000	1.000
	$\rho(x, Y)$	1.000	1.000	1.000
Consumption (C)	$\sigma(x)$	0.304	0.254	0.172
	$\sigma(x)/\sigma(Y)$	0.447	0.454	0.485
	$\rho(x, Y)$	0.774	0.689	0.644
Investment (I)	$\sigma(x)$	2.752	2.367	1.527
	$\sigma(x)/\sigma(Y)$	4.042	4.237	4.293
	$\rho(x, Y)$	0.953	0.942	0.927
Employment (N)	$\sigma(x)$	0.612	0.428	0.231
	$\sigma(x)/\sigma(Y)$	0.898	0.767	0.651
	$\rho(x, Y)$	0.949	0.941	0.875

Notes: $\sigma(x)$ denotes the standard deviation (in %), $\sigma(x)/\sigma(Y)$ is the relative standard deviation, and $\rho(x, Y)$ is the correlation with output. Higher σ reduces overall volatility.

Table 15 presents the sensitivity analysis for the risk parameter σ. The results show that higher values of σ systematically reduce volatility across all macroeconomic variables. Output volatility decreases from 0.681% to 0.356% as σ increases from 0.10 to 0.16. Investment remains the most volatile component, with relative volatility around 4 times that of output. The correlation patterns remain strong, though consumption's correlation with output weakens slightly at higher risk levels.

Figure 11 illustrates the impulse response functions corresponding to different risk parameter values. The dampening effect of higher σ is clearly visible across all variables, with the responses becoming more muted as risk aversion increases.

The price adjustment parameter analysis in Table 16 reveals that price flexibility has a moderate but consistent impact on business cycle dynamics. More flexible prices (lower π_d) generate higher output volatility, increasing from 0.665% to 0.694% as π_d decreases from 0.35 to 0.30. Employment shows the most sensitivity to price adjustment speed, with its correlation with output strengthening as prices become more flexible.

The impulse response analysis in Figure 12 confirms that greater price flexibility amplifies the economy's response to shocks, consistent with the volatility patterns observed in the business cycle statistics.

Table 17 examines the role of productivity shock variance. Interestingly, higher productivity shock variance σ_ϵ reduces output volatility while substantially increasing mean output levels.

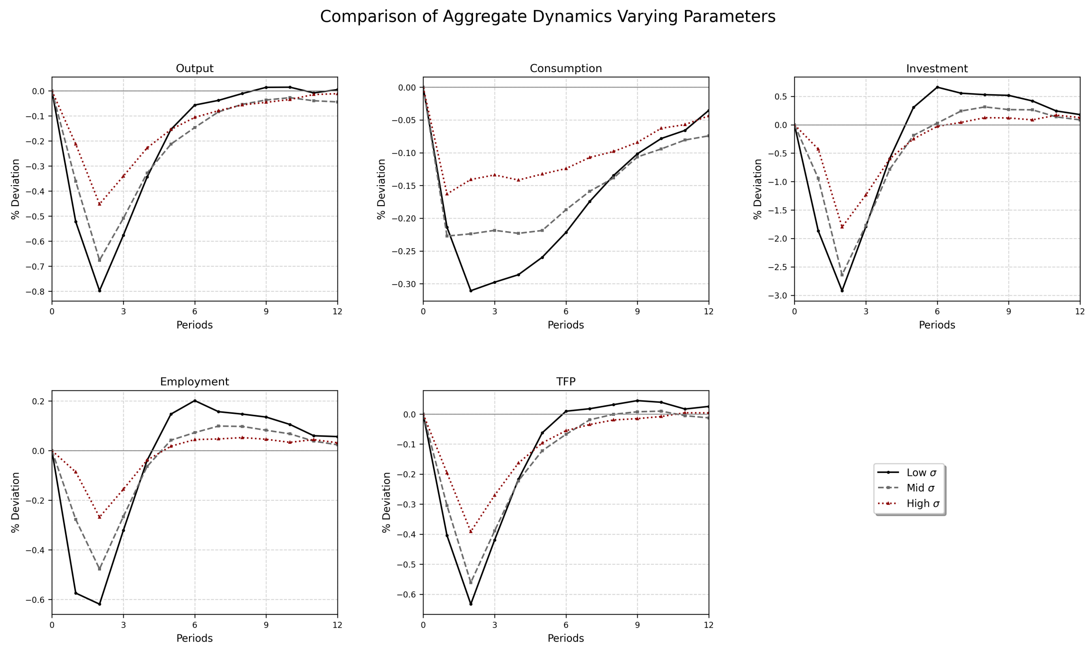


Figure 11: Impulse Response Functions: Varying σ (Risk Parameter)

Notes: Impulse responses to aggregate shocks across different risk parameter values. Higher σ dampens the response magnitude across all variables.

Table 16: Business Cycle Statistics: Varying Price Adjustment Parameter (π_d)

Variable	**Statistic**	**imp1658** ($\pi_d = 0.30$)	**imp2055** ($\pi_d = 0.325$)	**imp0051** ($\pi_d = 0.35$)
Output (Y)	$\sigma(x)$	0.694	0.679	0.665
	$\sigma(x)/\sigma(Y)$	1.000	1.000	1.000
	$\rho(x, Y)$	1.000	1.000	1.000
Consumption (C)	$\sigma(x)$	0.314	0.310	0.301
	$\sigma(x)/\sigma(Y)$	0.451	0.456	0.453
	$\rho(x, Y)$	0.772	0.772	0.759
Investment (I)	$\sigma(x)$	2.795	2.729	2.701
	$\sigma(x)/\sigma(Y)$	4.025	4.017	4.064
	$\rho(x, Y)$	0.952	0.951	0.950
Employment (N)	$\sigma(x)$	0.582	0.590	0.603
	$\sigma(x)/\sigma(Y)$	0.838	0.869	0.907
	$\rho(x, Y)$	0.921	0.937	0.955

Notes: Lower π_d indicates more flexible price adjustment. More flexible prices (lower π_d) lead to higher output volatility.

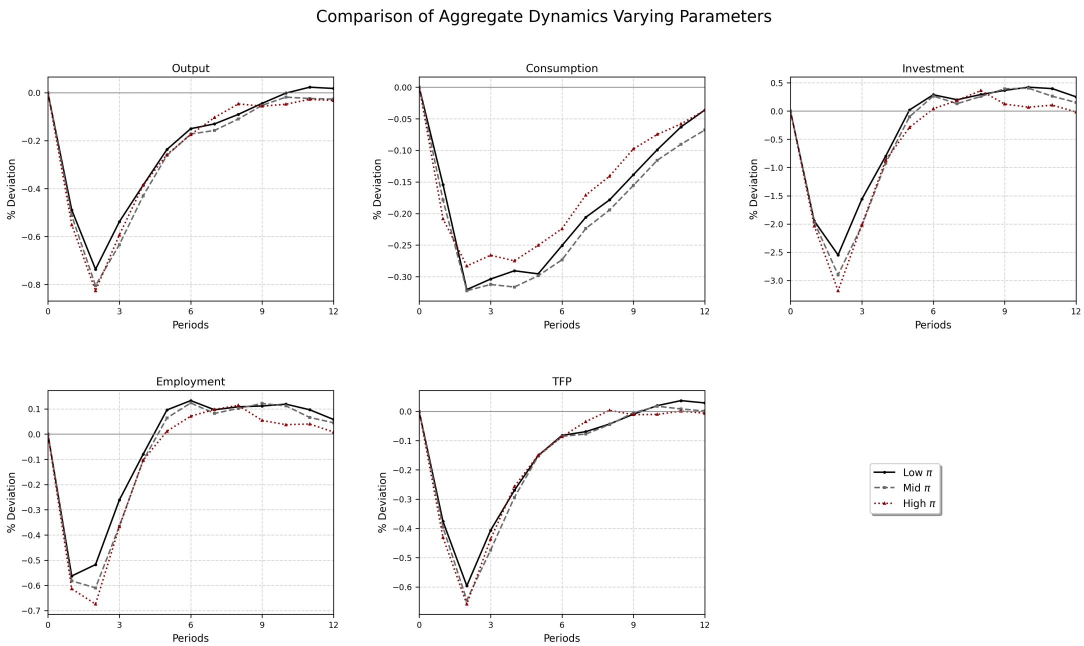


Figure 12: Impulse Response Functions: Varying π_d (Price Adjustment)

Notes: Impulse responses with varying price flexibility. Lower π_d increases response amplitudes in aggregate variables.

Table 17: Business Cycle Statistics: Varying Productivity Shock Parameter (σ_ϵ)

Variable	**Statistic**	**imp1712** ($\sigma_\epsilon = 0.46$)	**imp0035** ($\sigma_\epsilon = 0.48$)	**imp0438** ($\sigma_\epsilon = 0.49$)
Output (Y)	$\sigma(x)$	0.695	0.685	0.658
	$\sigma(x)/\sigma(Y)$	1.000	1.000	1.000
	$\rho(x, Y)$	1.000	1.000	1.000
Consumption (C)	$\sigma(x)$	0.314	0.304	0.282
	$\sigma(x)/\sigma(Y)$	0.452	0.444	0.429
	$\rho(x, Y)$	0.808	0.800	0.788
Investment (I)	$\sigma(x)$	2.742	2.736	2.672
	$\sigma(x)/\sigma(Y)$	3.945	3.993	4.063
	$\rho(x, Y)$	0.956	0.957	0.959
Employment (N)	$\sigma(x)$	0.645	0.683	0.681
	$\sigma(x)/\sigma(Y)$	0.928	0.997	1.035
	$\rho(x, Y)$	0.941	0.945	0.954

Notes: Higher σ_ϵ increases mean output levels substantially ($0.809 \rightarrow 0.849$) while maintaining strong correlations.

This suggests that larger productivity shocks create stabilizing effects through improved long-run growth prospects, despite the increased short-term uncertainty.

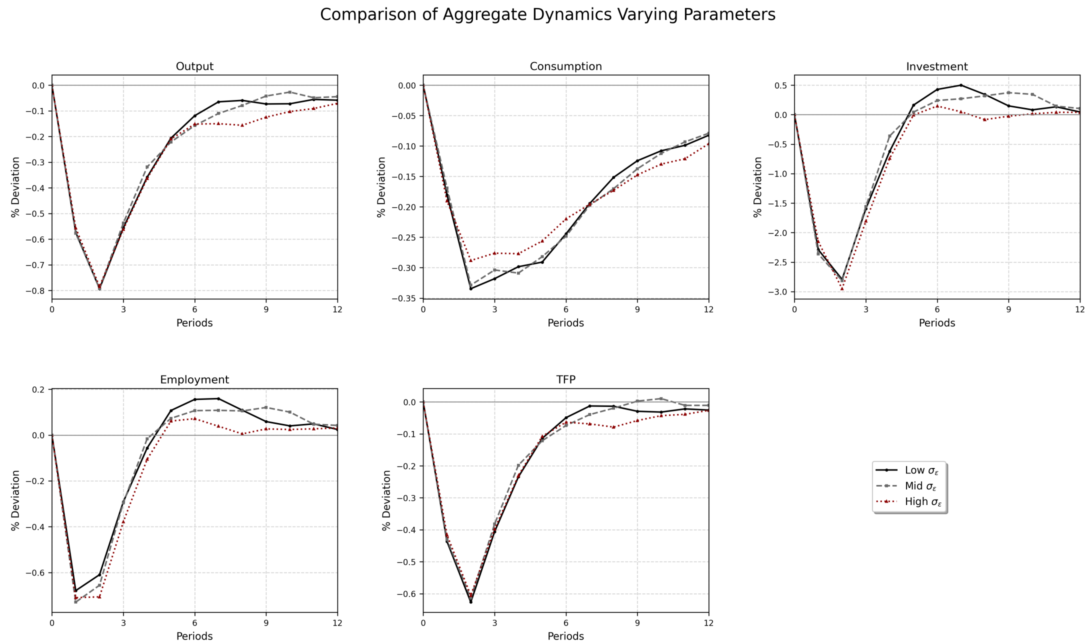


Figure 13: Impulse Response Functions: Varying σ_ϵ (Productivity Shock)

Notes: Responses to productivity shocks vary with σ_ϵ. Higher σ_ϵ increases mean output and stabilizes fluctuations.

The corresponding impulse responses in Figure 13 illustrate how higher productivity shock variance leads to more stable economic dynamics over time, reinforcing the stabilization mechanism identified in the business cycle statistics.

The capital share parameter α exhibits the strongest effects among all parameters examined, as shown in Table 18. Higher capital shares dramatically increase both output levels and volatility, with output volatility rising by 19.1% as α increases from 0.25 to 0.29. Notably, consumption's correlation with output weakens significantly, suggesting that higher capital intensity alters the fundamental co-movement patterns in the economy.

Figure 14 demonstrates the amplified responses associated with higher capital shares, confirming the increased sensitivity to shocks documented in the business cycle statistics.

Table 19 reveals highly non-monotonic effects of the upper bound parameter ξ_{ub}. Intermediate values (0.075 and 0.10) create severe distortions in consumption and investment behavior, with consumption volatility increasing dramatically and correlations with output collapsing. However, at $\xi_{ub} = 0.15$, stability is restored, suggesting the existence of threshold effects in the constraint mechanism.

Figure 15 illustrates the complex dynamics associated with different upper bound parameter values, with intermediate values generating the most volatile and unstable responses.

Table 20 provides a comprehensive summary of the parameter sensitivity analysis. The

Table 18: Business Cycle Statistics: Varying Capital Share Parameter (α)

Variable	**Statistic**	**imp2155** ($\alpha = 0.25$)	**imp0514** ($\alpha = 0.27$)	**imp1251** ($\alpha = 0.29$)
Output (Y)	$\sigma(x)$	0.701	0.732	0.835
	$\sigma(x)/\sigma(Y)$	1.000	1.000	1.000
	$\rho(x, Y)$	1.000	1.000	1.000
Consumption (C)	$\sigma(x)$	0.318	0.323	0.320
	$\sigma(x)/\sigma(Y)$	0.454	0.442	0.384
	$\rho(x, Y)$	0.794	0.733	0.669
Investment (I)	$\sigma(x)$	2.783	2.878	3.287
	$\sigma(x)/\sigma(Y)$	3.972	3.934	3.937
	$\rho(x, Y)$	0.954	0.952	0.963
Employment (N)	$\sigma(x)$	0.626	0.637	0.731
	$\sigma(x)/\sigma(Y)$	0.893	0.870	0.875
	$\rho(x, Y)$	0.939	0.944	0.965

Notes: Higher capital share α dramatically increases both output levels ($0.796 \rightarrow 0.910$) and volatility (+19.1%). Consumption correlation with output weakens significantly.

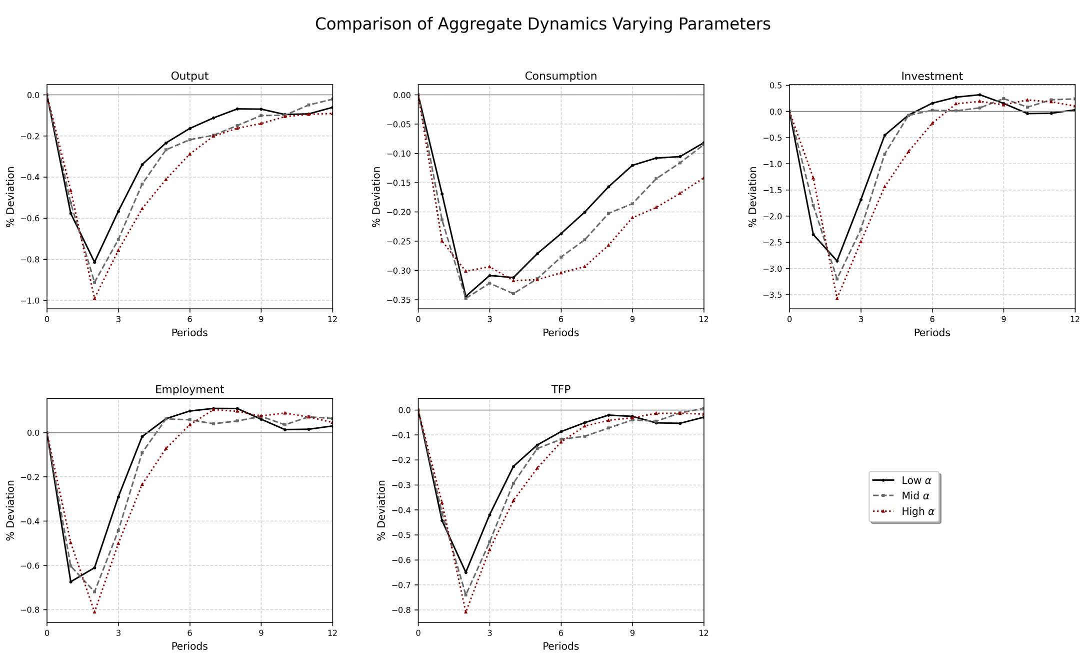


Figure 14: Impulse Response Functions: Varying α (Capital Share)
Notes: Changes in capital share alter the robustness of economic responses to shocks. Higher α amplifies volatility and growth.

Table 19: Business Cycle Statistics: Varying Upper Bound Parameter (ξ_{ub})

Variable	**Statistic**	**imp0051** ($\xi_{ub} = 0.00$)	**imp0824** ($\xi_{ub} = 0.075$)	**imp0320** ($\xi_{ub} = 0.10$)	**imp1519** ($\xi_{ub} = 0.15$)
	$\sigma(x)$	0.665	0.843	0.857	0.654
Output (Y)	$\sigma(x)/\sigma(Y)$	1.000	1.000	1.000	1.000
	$\rho(x, Y)$	1.000	1.000	1.000	1.000
	$\sigma(x)$	0.301	0.974	1.468	0.397
Consumption (C)	$\sigma(x)/\sigma(Y)$	0.453	1.155	1.714	0.607
	$\rho(x, Y)$	0.759	0.302	0.155	0.906
	$\sigma(x)$	2.701	5.618	7.770	2.311
Investment (I)	$\sigma(x)/\sigma(Y)$	4.064	6.658	9.065	3.533
	$\rho(x, Y)$	0.950	0.620	0.489	0.930
	$\sigma(x)$	0.603	0.761	0.768	0.512
Employment (N)	$\sigma(x)/\sigma(Y)$	0.907	0.902	0.896	0.783
	$\rho(x, Y)$	0.955	0.971	0.986	0.946

Notes: ξ_{ub} represents the upper bound constraint in the model. Intermediate values (0.075, 0.10) create significant distortions in consumption volatility and correlations, while $\xi_{ub} = 0.15$ restores stability.

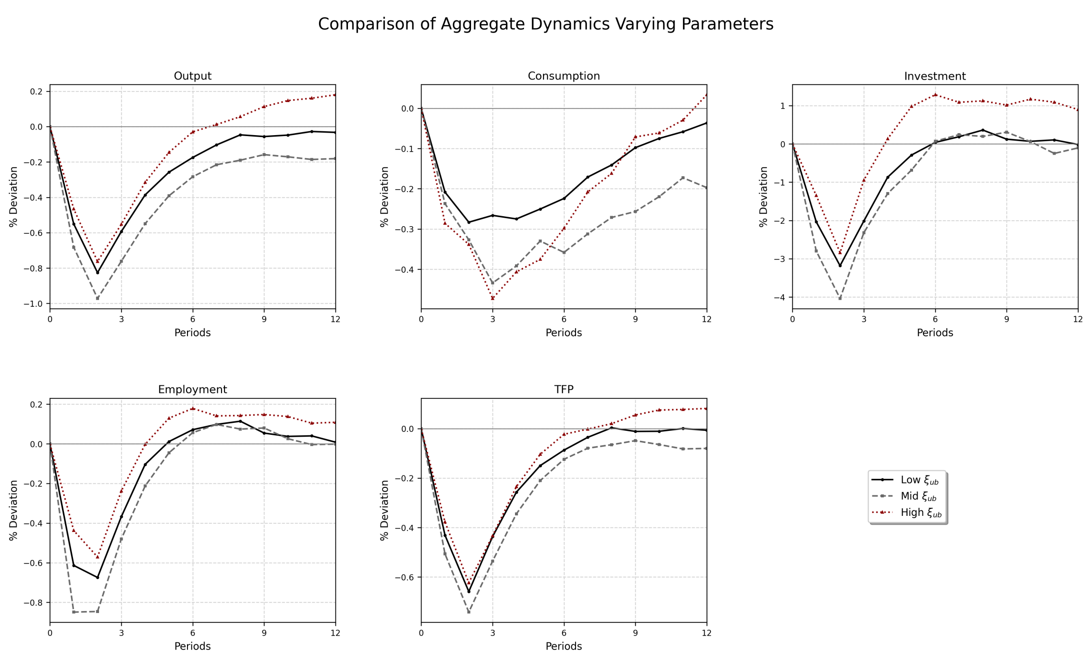


Figure 15: Impulse Response Functions: Varying ξ_{ub} (Upper Bound Parameter)

Notes: Non-monotonic effects of ξ_{ub} on economic dynamics. Intermediate values create volatility and reduce correlations, suggesting threshold effects in the constraint mechanism.

Table 20: Summary of Parameter Effects on Key Business Cycle Statistics

Parameter	**Range**	**Mean Output**	**Output Vol.**	**Inv. Vol.**	**C-Y Corr.**
σ	0.10 – 0.16	0.797 – 0.790	0.681 – 0.356	4.04 – 4.29	0.774 – 0.644
π_d	0.30 – 0.35	0.806 – 0.798	0.694 – 0.665	4.03 – 4.06	0.772 – 0.759
σ_ϵ	0.46 – 0.49	0.809 – 0.849	0.695 – 0.658	3.95 – 4.06	0.808 – 0.788
α	0.25 – 0.29	0.796 – 0.910	0.701 – 0.835	3.97 – 3.94	0.794 – 0.669

Notes: This table summarizes the range of effects across parameter variations. α has the strongest effects on both output levels and volatility, while σ primarily affects volatility.

capital share parameter α emerges as the most influential, affecting both output levels and volatility substantially. In contrast, the risk parameter σ primarily affects volatility without significant level effects, while the productivity shock parameter σ_ϵ has notable effects on output levels with stabilizing effects on volatility.

E Economic Responses to Increased Volatility

E.1 Partial Equilibrium Analysis

This section analyzes the partial equilibrium impulse response functions (IRFs) shown in Figure 16, which illustrate the economic responses to an increase in productivity volatility (σ_ε^2) for different values of the productivity reset probability π. Under partial equilibrium, wages and interest rates remain fixed, allowing us to isolate the direct effects of volatility changes on firm behavior.

To understand how the volatility shock propagates through the economy, we first examine the evolution of firm-level statistics. When the volatility shock hits in period 1, no immediate change occurs in the distribution of beliefs because firms have not yet drawn new productivity values. However, in period 2, both $\mathrm{Var}(\tilde{\varepsilon})$ and average uncertainty $\tilde{v}$ jump sharply as a fraction of firms draw from the more dispersed productivity distribution. Unlike a Bayesian uncertainty shock where belief compression reduces $\mathrm{Var}(\tilde{\varepsilon})$, here the variance of beliefs increases because resetting firms now have more dispersed true productivity values to learn about. This increased dispersion drives positive aggregate effects through the mechanisms identified in Proposition 1.

Following Proposition 1, aggregate output dynamics are driven by three key effects when productivity volatility increases. First, the Jensen effect arises because productivity $z = e^\varepsilon$ is log-normally distributed. Second, the Oi-Hartman-Abel (OHA) effect operates through the convexity of the marginal product of capital in firm-level productivity. Third, the reallocation effect emerges as greater productivity dispersion across firms creates larger potential output gains when capital is allocated efficiently.

When σ_ε^2 increases in period 1, all firms rationally anticipate that with probability π, they will draw a new productivity from the more dispersed distribution in the next period.[30] As shown in Figure 16, the immediate effects in period 1 are remarkably small. Output, employment, and TFP show virtually no response because the actual productivity distribution has not yet changed—only expectations about future draws have been affected. Investment exhibits a slight positive response, reflecting firms' forward-looking behavior. This modest increase arises because firms anticipate the positive effects identified in Proposition 1: the Jensen effect on mean productivity, the OHA effect on optimal capital stocks, and the reallocation effect from increased productivity dispersion. Notably, unlike Bayesian uncertainty shocks to σ^2, volatility shocks to σ_ε^2 do not generate negative uncertainty effects that compress belief dispersion, explaining why even the anticipation effect on investment is positive rather than negative.

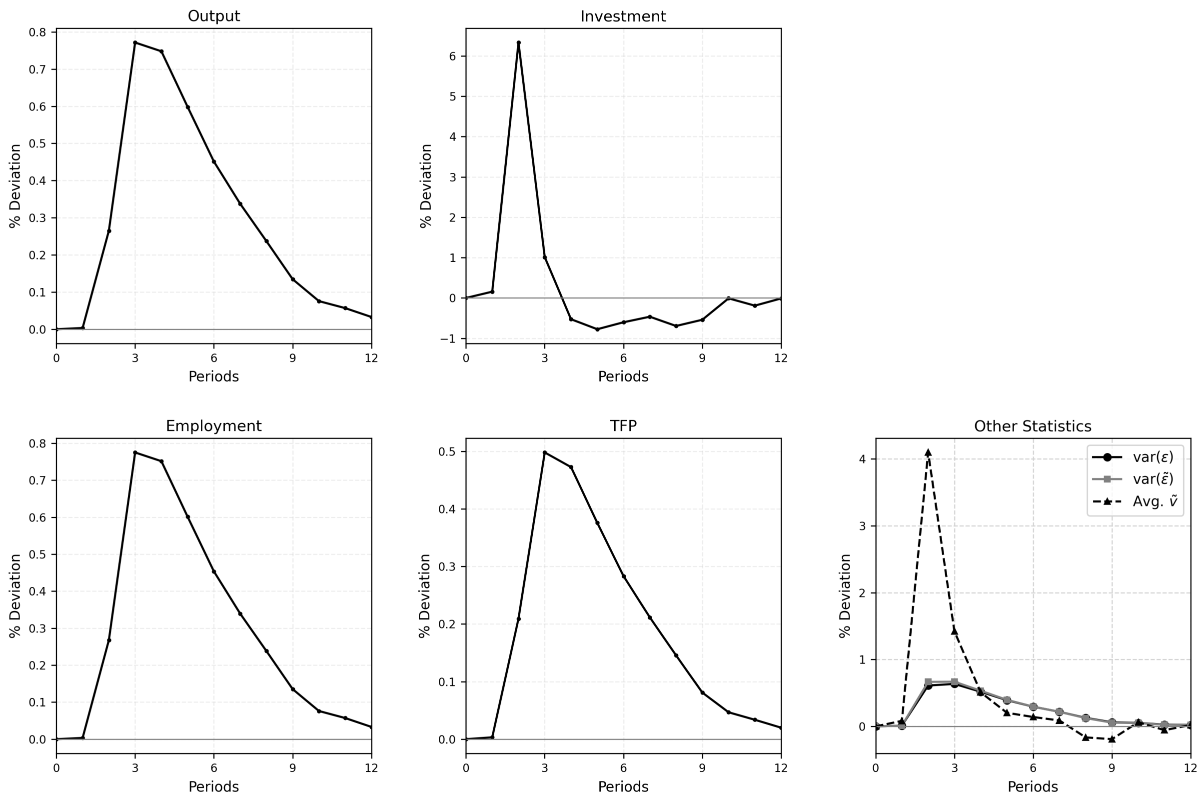


Figure 16: Economic Responses to Increased Volatility: Partial Equilibrium Analysis. The figure shows impulse response functions for output, employment, investment, and TFP following a permanent increase in productivity volatility σ_ε^2. Different lines represent varying productivity reset probabilities (π). Higher reset probabilities lead to larger and faster responses, as more firms draw from the new, more dispersed productivity distribution.

[30]From the Bellman equation (9), the volatility shock affects firms' expectations about future states. While the expected posterior mean $\mathbb{E}_{\sigma'|\sigma}[\tilde{\varepsilon}_0]$ remains unchanged (as the unconditional mean $\bar{\varepsilon}$ is unaffected), firms anticipate that if they reset, their initial uncertainty $\tilde{v}_0$ will be higher due to the increased σ_ε^2. This leads to higher expected capital choice k' through the convexity of optimal policies, but these effects only materialize in period 2 when productivity resets actually occur.

The dynamics change dramatically in period 2 when the volatility shock materializes. A fraction π of firms draw new productivity realizations from the more dispersed distribution and begin new learning cycles with more dispersed initial priors. The three theoretical effects now become evident in aggregate variables. The Jensen effect increases mean productivity for resetting firms, while the OHA effect induces higher investment due to return convexity. Most importantly, the reallocation effect improves allocative efficiency through greater productivity dispersion. All variables exhibit positive responses that gradually decay toward steady state as the economy adjusts to the new volatility regime.

The reset probability π plays a crucial role in shaping both the magnitude and timing of responses. Higher values of π generate larger peak responses as more firms are affected by the new productivity distribution. The effects also materialize more quickly when more firms reset, though all parameterizations eventually converge to similar steady states. This pattern confirms that the aggregate impact scales directly with the fraction of firms experiencing productivity resets (see Figure 17).

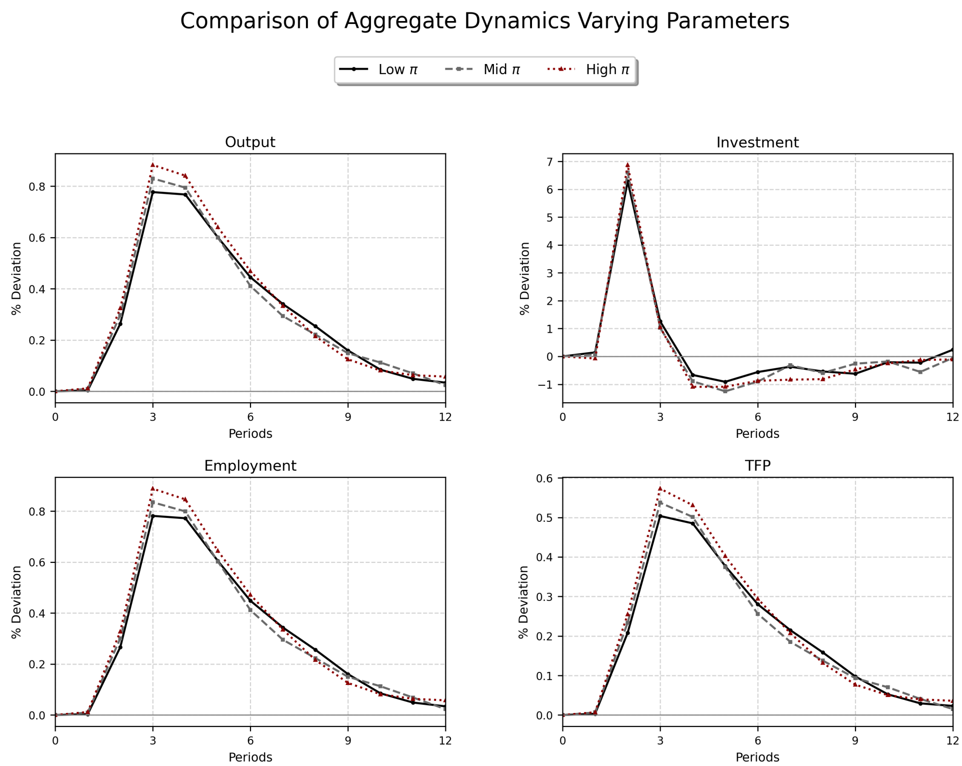


Figure 17: Economic Responses to Increased Volatility: Partial Equilibrium Analysis. The figure shows impulse response functions for output, employment, investment, and TFP following a permanent increase in productivity volatility σ_ε^2. Different lines represent varying productivity reset probabilities (π). Higher reset probabilities lead to larger and faster responses, as more firms draw from the new, more dispersed productivity distribution.

E.2 General Equilibrium Analysis

Figure 18 presents the general equilibrium impulse response functions following an increase in productivity volatility σ_ε^2. In contrast to the partial equilibrium analysis, these responses incorporate endogenous adjustments in wages and interest rates, as well as household consumption and savings decisions.

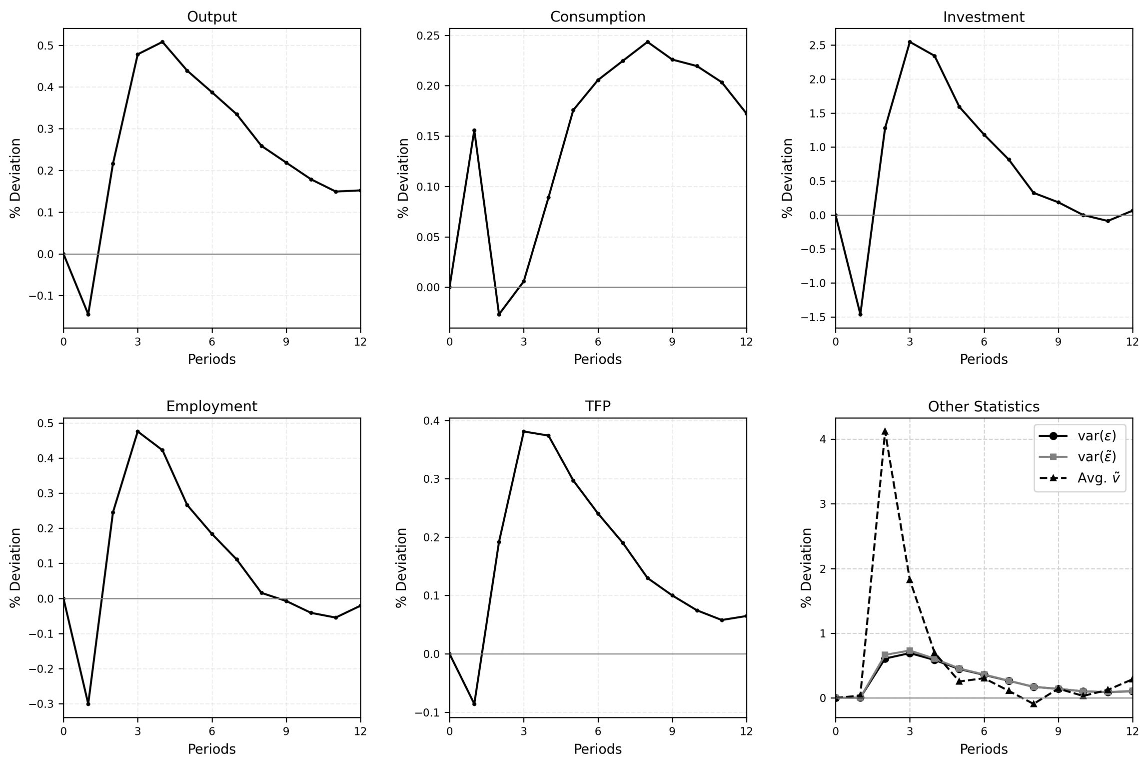


Figure 18: Economic Responses to Increased Volatility: General Equilibrium

Notes: Each panel plots the aggregate economy's response to an increase in volatility σ_ε^2, which occurs in period 1. The percent deviation from the variable's pre-shock average level is shown. The model incorporates endogenous factor prices and household optimization.

The general equilibrium responses reveal several important differences from the partial equilibrium case. Most strikingly, investment falls in period 1 under general equilibrium, contrasting with the slight increase observed under partial equilibrium. This reversal occurs because when all firms simultaneously anticipate higher future returns from the Jensen, OHA, and reallocation effects, their collective desire to increase investment drives up the equilibrium interest rate. This endogenous interest rate increase makes current investment more expensive, overwhelming the positive direct effects and causing aggregate investment to decline. Employment also contracts slightly as wage adjustments reflect firms' forward-looking behavior regarding future productivity dispersion.

The consumption dynamics provide further insight into the general equilibrium mechanisms. Consumption rises immediately in period 1, reaching approximately 0.15% above steady state.

This increase occurs because the decline in investment frees up resources for current consumption with output relatively unchanged in period 1 (since firms have not yet drawn new productivity). Additionally, households optimally choose to increase consumption in anticipation of higher future income from productivity gains. After a brief dip in period 2 as investment rebounds, consumption settles at a persistently higher level, reflecting the economy's enhanced productive capacity.

The contrast with Bayesian uncertainty shocks is particularly instructive. While noise shocks (σ^2) create belief compression that reduces $\text{Var}(\tilde{\varepsilon})$ and causes negative aggregate effects, volatility shocks (σ_z^2) work through an entirely different channel. Here, the variance of beliefs increases rather than decreases, as firms that reset face genuinely more dispersed productivity draws. This increased dispersion generates positive effects through all three channels in Proposition 1: the Jensen effect raises mean productivity, the OHA effect increases optimal capital choices, and the reallocation effect improves allocative efficiency, leading to the expansion observed in all real variables after period 2.