

Online Appendix

A Analytical model

A.1 Proof Lemma 1

The firm's objective at time t is to maximize its present discounted expected future profits by choosing an investment plan $\{i_{t+j}\}_{j=0}^{\infty}$:

$$\mathbb{E}_t \left(\sum_{j=0}^{\infty} \frac{1}{(1+r)^j} (\varepsilon_{i,t+j} k_{i,t+j}^{\alpha} - i_{i,t+j}) \right) \quad \text{s.t.} \quad k_{i,t+j+1} = (1-\delta)k_{i,t+j} + i_{i,t+j}, \quad \text{given } k_{i,t}, \quad (29)$$

where r is the interest rate faced by the firm.

By log-linearizing the F.O.C., we obtain that the decision rule is an increasing function of expected idiosyncratic productivity:

$$\ln(k_{i,t+1}) = \frac{1}{1-\alpha} \ln(\mathbb{E}_t(\varepsilon_{i,t+1})) - \frac{1}{1-\alpha} \ln\left(\frac{r+\delta}{\alpha}\right) \quad (30)$$

Given the log-linearity of the firms' FOC and the log-normality of the exogenous stochastic process $\ln(\varepsilon) \sim \mathcal{N}(\bar{\mu}, \sigma_{\mu}^2 + \sigma_{\varepsilon}^2)$, $\ln(k) \sim \mathcal{N}(\mathbb{E}(\ln(k)), \mathbb{V}(\ln(k)))$ where:

$$\mathbb{E}(\ln(k)) = \frac{1}{1-\alpha} \left(\bar{\mu} + \frac{\sigma_{\varepsilon}^2 + \sigma_{\mu}^2}{2} \right) - \frac{1}{1-\alpha} \ln\left(\frac{r+\delta}{\alpha}\right) \quad (31)$$

$$\mathbb{V}(\ln(k)) = \frac{\mathbb{V}(\mathbb{E}(\varepsilon))}{(1-\alpha)^2} = \frac{\sigma_{\mu}^2}{(1-\alpha)^2} \quad (32)$$

Accordingly, $(\ln(\varepsilon), \ln(k))$ are jointly normally distributed where the covariance is as follows:

$$\text{Cov}(\ln(\varepsilon), \ln(k)) = \mathbb{E}(\ln(k) \ln(\varepsilon)) - \mathbb{E}(\ln(k))\mathbb{E}(\ln(\varepsilon)) = \frac{\sigma_{\mu}^2}{(1-\alpha)} \quad (33)$$

where

$$\begin{aligned}\mathbb{E}(\ln(k) \ln(\varepsilon)) &= \frac{1}{1-\alpha} \mathbb{E} \left[(\mu + \ln(e)) \left(\mu + \frac{\sigma_\mu^2 + \sigma_e^2}{2} - \ln \left(\frac{r+\delta}{\alpha} \right) \right) \right] \\ &= \frac{1}{1-\alpha} \mathbb{E}(\mu^2) + \frac{\bar{\mu}}{1-\alpha} \left[\frac{\sigma_\mu^2 + \sigma_e^2}{2} - \ln \left(\frac{r+\delta}{\alpha} \right) \right]\end{aligned}\tag{34}$$

$$\mathbb{E}(\ln(k)) \mathbb{E}(\ln(\varepsilon)) = \frac{1}{1-\alpha} \bar{\mu} \left[\bar{\mu} + \frac{\sigma_\mu^2 + \sigma_e^2}{2} - \ln \left(\frac{r+\delta}{\alpha} \right) \right]\tag{35}$$

A.2 Proof Proposition 1

Given the idiosyncratic output follows $y \sim \mathcal{LN}(\mu_y, \sigma_y)$, $\frac{y_i}{N} \sim \mathcal{LN}(\mu_y - \ln(N), \sigma_y)$. By the Fenton-Wilkinson approximation, we can approximate the sum of i.i.d. log-normal distributions as follows¹⁶

$$\sum_{i=1}^N \frac{y_i}{N} \sim \mathcal{LN}(\mu_Y, \sigma_Y)\tag{36}$$

where:

$$\begin{aligned}\mu_Y &= \mu_y + \frac{\sigma_y^2}{2} - \frac{\sigma_Y^2}{2} \\ \sigma_Y^2 &= \ln \left[\frac{e^{\sigma_y^2} - 1}{N} + 1 \right]\end{aligned}$$

Accordingly we can approximate the expected value, $\mathbb{E}(Y)$, and variance, $\mathbb{V}(Y)$, as follows

$$\begin{aligned}\mathbb{E}(Y) &= e^{\mu_y + \sigma_y^2} \\ \mathbb{V}(Y) &= \frac{e^{\sigma_y^2} - 1}{N} e^{2\mu_y + \sigma_y^2}\end{aligned}\tag{37}$$

B Frictionless Economy

In this Appendix, we show that the endogenous aggregate μ can be exactly characterized by the first moment of the marginal distribution of capital K and the dynamic productivity distribution h with $\psi = 0$.

¹⁶See Marlow (1967).

In equation (11), the choice of the current level of employment can be derived from a static problem as:

$$N(\varepsilon, k; \mu) = \arg \max_n [\varepsilon k^\alpha n^\nu - \omega(\mu)n] \quad (38)$$

which yields

$$N(\varepsilon, k; \mu) = [\nu \varepsilon k^\alpha / \omega(\mu)]^{1/(1-\nu)} \quad (39)$$

Using this decision rule for employment, we can replace the first and second terms in equation (11) as:

$$\varepsilon k^\alpha n^\nu - \omega(\mu)n = (1 - \nu) \varepsilon^{1/(1-\nu)} k^{\alpha/(1-\nu)} \left(\frac{\nu}{\omega(\mu)} \right)^{\nu/(1-\nu)}, \quad (40)$$

and we can rewrite the problem as follows:

$$\begin{aligned} v(\varepsilon, k; \mu) = \max_{k'} & \left[(1 - \nu) \varepsilon^{1/(1-\nu)} k^{\alpha/(1-\nu)} \left(\frac{\nu}{\omega(\mu)} \right)^{\nu/(1-\nu)} + (1 - \delta)k - k' \right. \\ & \left. + E[d(\mu, \mu')v(\varepsilon', k'; \mu') \mid \varepsilon, \mu] \right]. \end{aligned} \quad (41)$$

This problem yields the optimal investment decision $G(\varepsilon; \mu)$ as follows:

$$G(\varepsilon; \mu) = L_0(\varepsilon)L_1(\mu) \quad (42)$$

$$L_0(\varepsilon; \mu) = \left(\sum_{\mu'} \Pi^\mu(\mu'|\mu) \sum_{\varepsilon'} \Pi_{\mu'|\mu}^\varepsilon(\varepsilon'|\varepsilon) \varepsilon'^{1/(1-\nu)} \right)^{(1-\nu)/(1-(\alpha+\nu))} \quad (43)$$

$$L_1(\mu) = \left(\frac{1 - (1 - \delta) \sum_{\mu'} \Pi^\mu(\mu'|\mu) d(\mu, \mu')}{\alpha \sum_{\mu'} \Pi^\mu(\mu'|\mu) d(\mu, \mu') \left(\frac{\nu}{\omega(\mu')} \right)^{\nu/(1-\nu)}} \right)^{\frac{1-\nu}{\alpha+\nu-1}}. \quad (44)$$

This shows that the investment decision is independent of the current capital stock k , which depends only on the idiosyncratic productivity of the previous period. This implies that (1) it is sufficient to track the idiosyncratic productivity both in the current and previous periods for each firm and (2) the distribution of the current and previous idiosyncratic productivity h is a N_ε by N_ε grid point object.

It follows that the distribution of firms over idiosyncratic productivity and capital stock can be recovered, $\mu(\varepsilon_i(t), k_h(t))$, from h_t in each period t as follows. First, we can construct

$\bar{h}_{t-1}(\varepsilon_j)$, the marginal distribution of firms over ε_j for $j = 1, \dots, N_\varepsilon$ in $t - 1$, and $\bar{h}_t(\varepsilon_i)$, the marginal distribution of firms over ε_i for $i = 1, \dots, N_\varepsilon$ in t . We can also construct $\Pi_h^\varepsilon(\varepsilon_j, \varepsilon_i)$, the transition probability $Pr(\varepsilon_t = \varepsilon_i \mid \varepsilon_{t-1} = \varepsilon_j)$. Therefore, we can construct $\mu(\varepsilon, k)$, the distribution of firms over productivity and stock of capital in each period t as

$$\mu(\varepsilon_i, k_j) = \bar{h}_{t-1}(\varepsilon_j) \Pi_h^\varepsilon(\varepsilon_j, \varepsilon_i) \quad (45)$$

$$k_j = \frac{L_0(\varepsilon_j) \bar{h}_{t-1}(\varepsilon_j)}{\sum_m L_0(\varepsilon_m) \bar{h}_{t-1}(\varepsilon_m)} K_t \quad (46)$$

C Dynamic Distributions and its Probability Space

In this Section we describe the procedure throughout which we select the set of N_m moments \bar{m} of the marginal productivity distribution \bar{h} that we use to build the set of dynamic distribution H and its probability space. We then explain how we construct the set of dynamic distributions H^* .

C.1 Moment selection

We focus on the moments \bar{m} that are the most informative with regard to the fluctuation of \bar{h} . Consequently, we select the moments that have the highest volatility. To this end, we implement a procedure based on the following steps:

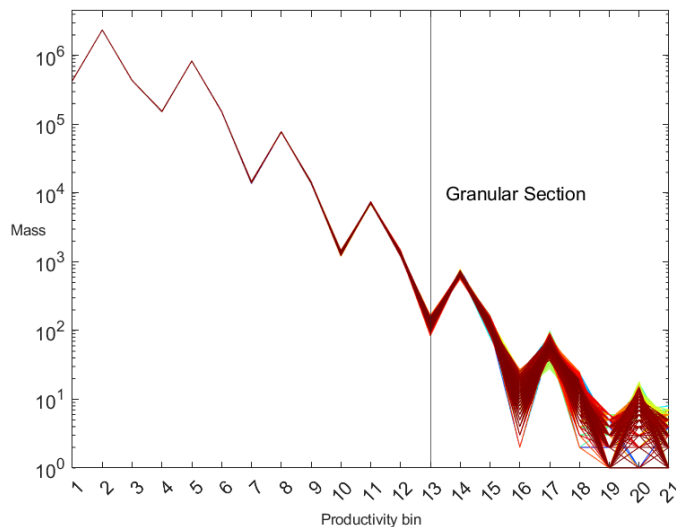
1. We identify the granular section of the productivity distribution. In other words, the part of the productivity distribution \bar{h} in which L.L.N. fails.
2. We select the set of most volatile moments that allow us to replicate the fluctuation of the firm distribution in the granular section.

Figure 3 reports the simulated productivity distributions \bar{h} on a logarithmic scale over 24,000 periods. The simulation shows that the L.L.N. does not hold on the right-tail of the productivity distribution. Consequently, we define the granular section of the productivity distribution as the part of \bar{h} that is associated with $\varepsilon \geq \varepsilon_{13}$.

As next step, in Figure 4, we report the distributions of the percentage deviation from its ergotic value of moments that can describe the fluctuations in the productivity distribution \bar{h} in the granular section. In particular, Figure 4 displays the percentage deviation of (A) mean productivity, (B) standard deviation of productivity, (C) Pearson moment coefficient of skewness, and (D) firm mass, calculated among firms $\varepsilon \geq \varepsilon_{13}$. Surprisingly, the simulation shows that the dispersion of the percentage deviation of the mean, 0.398, is relatively low compared to 5.655, 8.632, and 2.999, which are the percentage dispersions of the standard deviation, Pearson's moment coefficient of skewness, and mass of firms, respectively. Therefore, we simulate a granular economy that replicates the cyclicity of the (1) standard deviation, (2) the Pearson's moment coefficient of skewness, and (3) firm mass of the firm in the granular section of the productivity distribution.

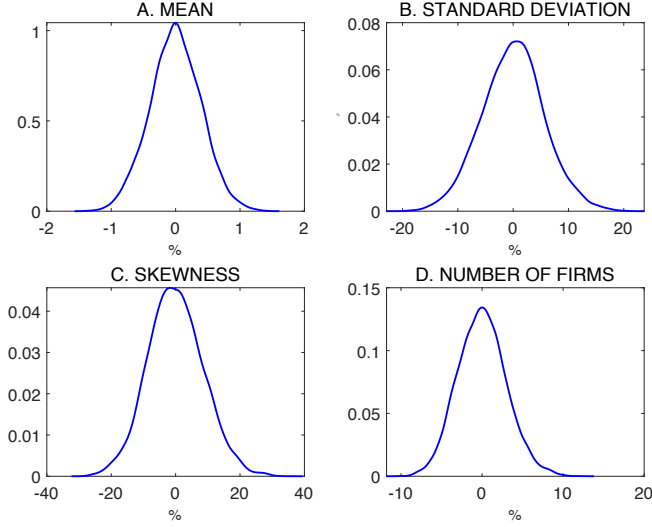
To discretize the multivariate stochastic process of the moment \bar{m} , we use three points on the grid, $d_m = 3$, for each moment.

Figure 3: Simulated Productivity Distribution \bar{h}



Notes: The figure shows the simulated productivity distribution \bar{h} on the log scale over the 24,000 periods. In particular, the figure reports the mass of firms for each productivity.

Figure 4: Moments of the Tail



Notes: The figure reports the distribution of the percentage deviations from its ergotic value of (A) mean productivity, (B) standard deviation of productivity, (C) the Pearson’s moment coefficient of skewness, and (D) firm mass, calculated among firms $\varepsilon \geq \varepsilon_{13}$. The distributions are obtained from a 24,000-period simulation of the productivity distribution \bar{h}

C.2 Building the Set of Dynamic Distribution H

After having obtained the discretized set of moments M , we proceed to the construction of the h . Our procedure is based on N_H iteration of the following two blocks in order to sample the set of the N_H sets of dynamic distribution $\mathbb{H} = \{H^1, H^2, \dots, H^{v-1}, H^v, H^{v+1}, \dots, H^{N_H-1}, H^{N_H}\}$:

1. We first recover the set of $N_{\bar{h}} = d_m^{N_m}$ marginal distributions of productivity, \bar{H}^v . To this end, we simulate the productivity distribution \bar{h} for T period to obtain a time series of the marginal productivity distribution, and then we select a subset of $d_m^{N_m-1}$ based on their standard deviation and skewness, such that $\forall j = 1, \dots, d_m^{N_m-1}$:

$$\bar{h}_j^v = \arg \min_{\bar{h}} | m_{1,j} - \text{st.dev.}(\bar{h}) | + | m_{2,j} - \text{skew}(\bar{h}) | \quad (47)$$

Finally, given the subset of productivity distribution, we build \bar{H}^v by re-scaling the mass on the granular section in order to match the third moment, m_3 .

2. Given Π^ε , we construct the set of dynamic productivity distributions H^v by imple-

menting a constrained draw that requires that the set of dynamic productivity

$$H^v = \{h_1^v, h_2^v, \dots, h_{l+(m-1)N_{\bar{h}}-1}^v, h_{l+(m-1)N_{\bar{h}}}^v, h_{l+(m-1)N_{\bar{h}}+1}^v, \dots, h_{N_{\bar{h}}^2-1}^v, h_{N_{\bar{h}}^2}^v\}$$

is such that $\forall l, m = 1, 2, \dots, N_{\bar{h}}$ and $\forall i, j = 1, 2, \dots, N_{\varepsilon}$

$$\bar{h}_l^v(\varepsilon_i) = \sum_j h_{l+(m-1)N_{\bar{h}}}^v(\varepsilon_i, \varepsilon_j)$$

$$\bar{h}_m^v(\varepsilon_j) = \sum_i h_{l+(m-1)N_{\bar{h}}}^v(\varepsilon_i, \varepsilon_j)$$

Finally we obtain the the set of productivity distributions H^* as the average across \mathbb{H} , such that:

$$h_f^* = \frac{\sum_v h_f^v}{N_H} \quad \forall f = 1, 2, \dots, N_{\bar{h}^2}$$

C.3 Solution Performance

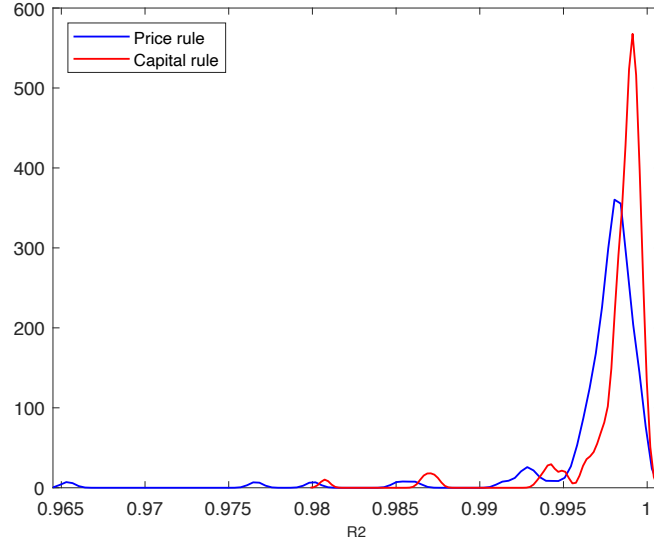
Our procedure to build dynamic time-varying transition probabilities delivers an error of 0.0002672, that is computed as follows:

$$\frac{1}{N_H} \sum_{j=1}^{N_H} \frac{1}{N_m} \left(\frac{|m_{1,j}^{target} - \text{st.dev.}(h^{j,*})|}{m_{1,j}^{target}} + \frac{|m_{2,j}^{target} - \text{skew}(h^{j,*})|}{m_{2,j}^{target}} + \frac{|m_{3,j}^{target} - \text{Tail Mass}(h^{j,*})|}{m_{3,j}^{target}} \right) \quad (48)$$

Figure 5 reports the kernel densities of the R^2 of the 136 forecasting rules of price and capital.

The mean of R^2 are 0.998 and 0.997 for the capital and price equation, respectively.

Figure 5: Distribution of the R^2 of the Forecasting Rules



Notes: The figure reports the kernel-densities of the R^2 of the 136 Price, blue line, and Capital, red line, forecasting rules distribution.

D Non-fully Rational Firms

In this Section, we present the results of the non-fully rational model. Firms form expectations for idiosyncratic productivity using the steady-state transition probability. Such as the recursive problem is:

$$V(\varepsilon, k; \mu) = \max_{n, k'} \varepsilon F(k, n) - \omega(\mu)n - i + \sum_{\mu'} \Pi^\mu(\mu' | \mu) d(\mu, \mu') \sum_{\varepsilon'} \Pi^\varepsilon(\varepsilon' | \varepsilon) V(\varepsilon', k'; \mu') \quad (49)$$

subject to

$$\hat{i} = k' - (1 - \delta)k \quad (50)$$

$$i = \hat{i}(1 - \psi \mathbb{1}_{\hat{i} < 0}) \quad (51)$$

$$k', n \in \mathcal{R}_+ \quad (52)$$

Table 7: Business Cycle Moments of Non-Fully Rational Model

	GE			PE		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x, y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x, y)$
$\ln Y$	0.155	1.000	1.000	0.342	1.000	1.000
$\ln C$	0.089	0.578	0.723	0.911	2.663	0.788
$\ln I$	0.674	4.360	0.892	2.884	8.434	-0.464
$\ln H$	0.109	0.705	0.825	0.342	1.000	1.000

Notes: The table shows the general (GE) and partial (PE) model equilibrium business cycle moments of output Y , consumption C , investment I , and hours worked H . $\sigma(x)$ is the standard deviation of x , and $\sigma(x)/\sigma(Y)$ is the relative standard deviation to that of Y , and $\rho(x, Y)$ is the contemporaneous correlation of x with Y .

E Empirics

E.1 Data Construction

Variables. We define the variables used in our empirical analysis as follows:

1. *Investment rate* is obtained by the ratio of capital expenditures (CAPX) to lagged plant, property, and equipment (PPEGT). We control for the 3-digit sector fix effect.
2. *Inaction rate* among the 100 largest firms of the previous period is defined as the mass of firms such that $|i/k| \leq 0.01$.
3. *Marginal product of capital* is given by the logarithm of the ratio of sales (SALE) to physical capital (PPEGT). We control for the 3-digit sector times year fix effect.

4. *Idiosyncratic shocks* are proxied in three different ways. The first proxy, the benchmark, is the productivity growth rate :

$$\Delta\varepsilon_{i,t}^1 = \frac{\varepsilon_{i,t} - \varepsilon_{i,t-1}}{\varepsilon_{i,t-1}} \quad (53)$$

The second proxy is the arc-percent change:

$$\Delta\varepsilon_{i,t}^2 = \frac{1}{2} \frac{\varepsilon_{i,t} - \varepsilon_{i,t-1}}{\varepsilon_{i,t} + \varepsilon_{i,t-1}} \quad (54)$$

We finally compute the third proxy as the residual of the following linear auto-regressive model:

$$\ln \varepsilon_{i,t} = \beta_0 + \beta_1 \ln \varepsilon_{i,t-1} + \Delta\varepsilon_{i,t}^3, \quad (55)$$

We control for 3-digit sector times year fix effect for all the three proxies.

5. *Granular residual* is obtained as the difference between the sales-weighted sum of the idiosyncratic shocks among the largest 100 firms of the previous period and the equal weighted shocks among the largest 100 firms of the previous period, such that $\forall j = 1, 2, 3$:

$$\Theta_{100,t}^j = \sum_{i=1}^{100} \frac{\text{Sale}_{i,t-1}}{\text{Sale}_{t-1}^{\text{top100}}} \Delta\varepsilon_{i,t}^j - \sum_{i=1}^{100} \Delta\varepsilon_{i,t}^j \quad (56)$$

6. *Granular skewness* is approximated by the median skewness of the shocks among the largest 100 firms of the previous period, such that $\forall j = 1, 2, 3$:

$$\Upsilon_{100}^j = \frac{3(\Delta\varepsilon_t^{\text{mean},j} - \Delta\varepsilon_t^{\text{median},j})}{\sigma_{\Delta\varepsilon,t}^j} \quad (57)$$

7. *Granular capital misallocation* is approximated with the dispersion of the marginal product of capital:

$$\Sigma_{100} = \sqrt{\frac{\sum_{i=1}^{100} (\text{mpk}_{i,t} - \text{mpk}_t^{\text{mean}})^2}{100}} \quad (58)$$

Sample Selection. Using Standard Industry Classification (SIC) codes, we exclude firms in the oil, energy, and financial sectors. Specifically, we exclude oil and oil-related firms

with SIC codes 2911, 5172, 1311, 4922, 4923, 4924, and 1389; energy firms with SIC codes between 4900 and 4940; and financial firms with SIC codes between 6000 and 6999. We eliminate sample firms with missing data to ensure that sales data are valid for all samples.

Table 8: Summary Statistics

x	Observations	mean	sd	min	max	$\rho(x, \ln(Y))$
Θ_{100}^1	34	0.000	1.736	-3.458	5.333	0.060
Θ_{100}^2	34	0.000	1.234	-2.135	3.771	0.084
Θ_{100}^3	34	0.000	1.162	-2.529	2.950	0.104
Υ_{100}^1	34	0.000	25.602	-53.787	38.135	0.108
Υ_{100}^2	34	0.000	26.064	-64.458	45.354	0.077
Υ_{100}^3	34	0.000	20.929	-44.202	33.974	0.200
$\ln(\Sigma_{100})$	35	0.000	4.597	-8.296	13.011	-0.415
$\ln(\Sigma_{-100})$	35	0.000	1.281	-2.588	2.606	0.355
Inaction rate	34	9.529	4.280	3.000	17.000	0.222

Notes: This table shows descriptive statistics and correlation with the logarithm of the real GDP (GDPCA from FRED) for variables used in Sections 3, 4.3, and E.2. The granular statistics, residual, Θ_{100} , skewness, Υ_{100} , and dispersion of the marginal product of capital, Σ_{100} , the dispersion of the marginal product of capital excluding the largest 100 firms of the previous period, Σ_{-100} , and the inaction rate, are defined in Appendix E.1. Inaction rate denotes the percentage of the mass of firms among the 100 largest firms of the previous period whose investment rate is below 1 percent in absolute value. All series, except for the inaction rate, are HP-filtered with a smoothing parameter of 6.25.

E.2 Robustness checks

This section contains additional robustness analysis referenced in the main text.

E.2.1 1 Percent Winsorization

In this section, we replicate the analysis of 4.3 when we winorize the productivity growth rate, the marginal product of capital among the 100 largest firms, and the marginal product of capital excluding the largest 100 firms at 1 percent.

Table 9: Summary Statistics

x	Observations	mean	sd	min	max	$\rho(x, \ln(Y))$
Θ_{100}^1	34	0.000	1.442	-2.869	4.243	0.061
Υ_{100}^1	34	0.000	25.587	-52.034	38.474	0.150
$\ln(\Sigma_{100})$	35	0.000	3.806	-8.252	7.946	-0.421
$\ln(\Sigma_{-100})$	35	0.000	1.270	-3.207	2.486	0.248

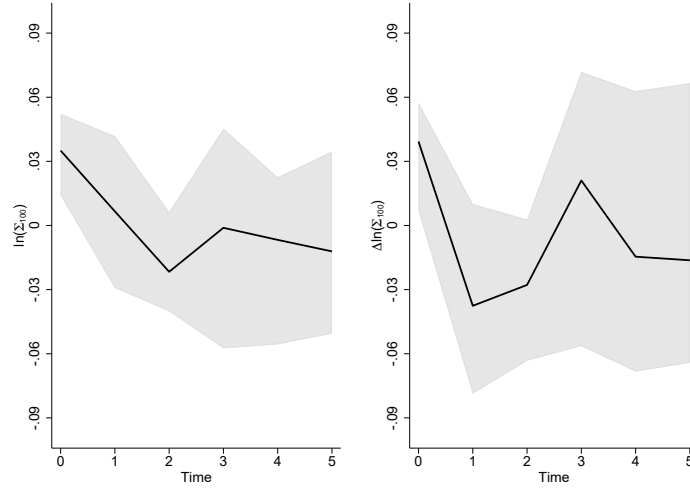
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Table 10: Cyclicity of the Granular Statistics with 1 Percent Winsorization

	$\Theta_{100,t}$	$\Upsilon_{100,t}$	$\ln(\Sigma_{100})$
ζ_t	0.192** (0.088)	0.014*** (0.004)	-0.084** (0.036)
ζ_{t-1}	0.293** (0.126)	0.015** (0.007)	0.007 (0.042)
ι_{t-1}	0.460** (0.174)	0.455** (0.174)	0.288 (0.215)
Observations	33	33	33
adj.R2	0.317	0.352	0.185
$\sigma(\zeta)$	1.442	25.587	3.806
$\rho(\zeta, \iota)$	0.061	0.150	-0.421

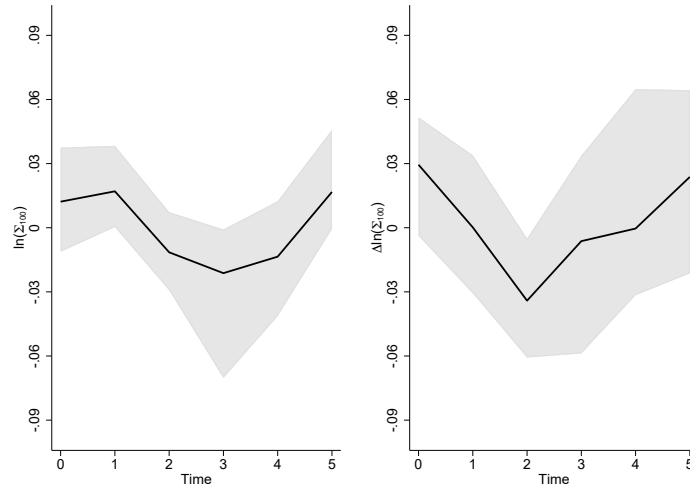
Notes: Results of estimating the regression $\iota_t = \beta_0 + \beta_1\zeta_t + \beta_2\zeta_{t-1} + \beta_3\iota_{t-1} + v_t$ where ι_t is the result of interest (the logarithm of the real GDP, GDPCA from FRED); ζ_t are the granular statistics (granular residual and skewness). All series are HP-filtered with a smoothing parameter of 6.25. Standard errors are given in parentheses. $\sigma(\zeta)$ and $\rho(\zeta, \iota)$ represent the standard deviation and the correlation with the logarithm of real GDP, respectively, for the period 1985-2019.

Figure 6: Effect of granular residual shock on granular misallocation.



Notes: Responses of the level and first difference of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular residual shock. The left figure shows the response of level, and the right figure shows the response of first difference of the logarithm of the granular dispersion of the marginal product of capital. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 519 repetitions.

Figure 7: Effect of granular skewness shock on granular misallocation.



Notes: Responses of the level and first difference of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular skewness shock. The left figure shows the response of level, and the right figure shows the response of first difference of the logarithm of the granular dispersion of the marginal product of capital. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 519 repetitions.

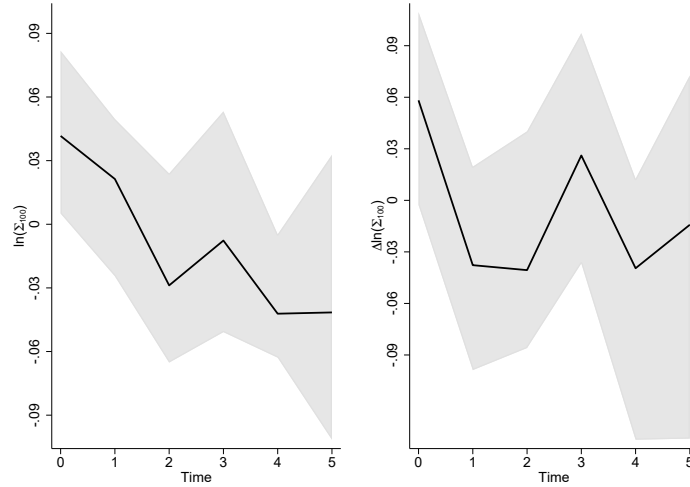
E.2.2 Alternative proxies of idiosyncratic shocks

Table 11: Cyclicalty of the Granular Statistics for alternative proxies of shocks

	$\Theta_{100,t}^2$	$\Theta_{100,t}^3$	$\Upsilon_{100,t}^2$	$\Upsilon_{100,t}^3$
ζ_t	0.218** (0.100)	0.234* (0.120)	0.017*** (0.004)	0.018** (0.009)
ζ_{t-1}	0.331** (0.136)	0.364** (0.159)	0.021*** (0.006)	0.013 (0.008)
ι_{t-1}	0.433** (0.170)	0.422** (0.160)	0.530*** (0.128)	0.455*** (0.150)
Observations	33	33	33	33
adj.R2	0.296	0.313	0.488	0.287
$\sigma(\zeta)$	1.234	26.064	1.162	20.929
$\rho(\zeta, \iota)$	0.084	0.077	0.104	0.200

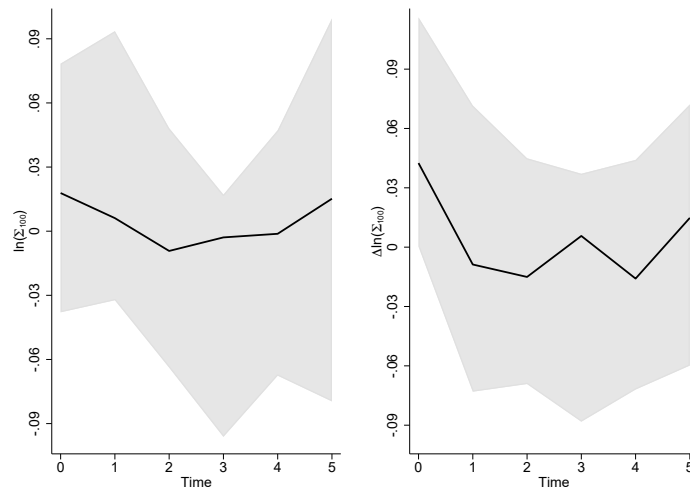
Notes: Results from estimating the regression $\iota_t = \beta_0 + \beta_1\zeta_t + \beta_2\zeta_{t-1} + \beta_3\iota_{t-1} + v_t$ where ι_t is the outcome of interest (the logarithm of the Real GDP, GDPCA from FRED); ζ_t is the granular statistics (granular residual and skewness). All series are HP-filtered with a smoothing parameter of 6.25. Standard errors are given in parentheses. $\sigma(\zeta)$ and $\rho(\zeta, \iota)$ represent the standard deviation of the granular shocks and their correlation with the real GDP, respectively for the period 1985-2019.

Figure 8: Effect of granular residual shock $\Theta_{100,t}^2$ on granular misallocation.



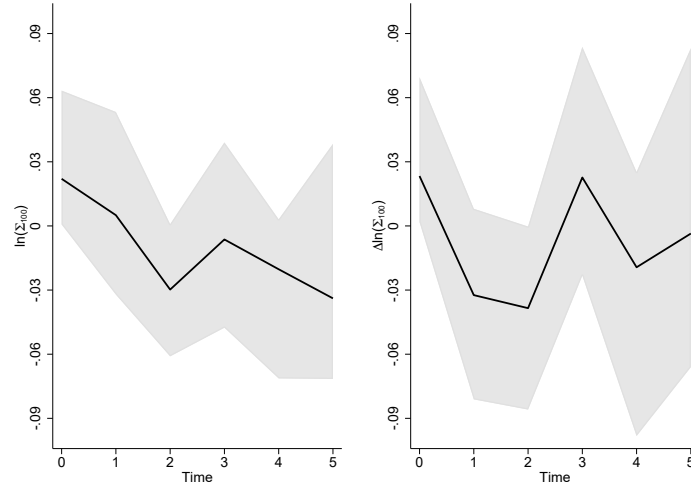
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Figure 9: Effect of granular skewness shock Υ_{100}^2 on granular misallocation.



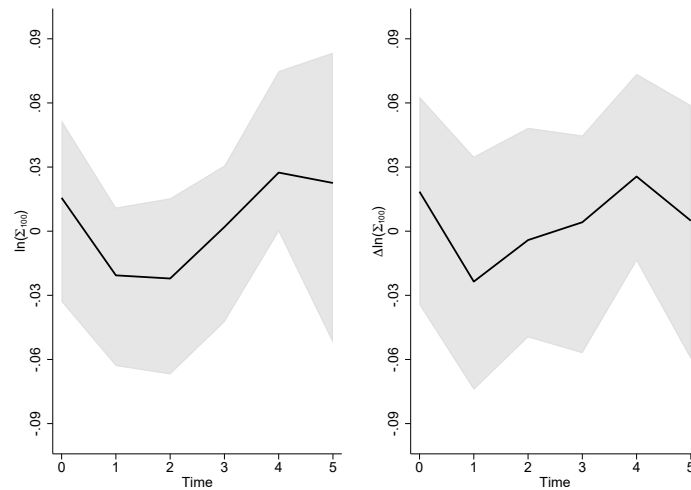
Notes: Responses of the level and first difference of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular skewness shock. The left figure shows the response of level, and the right figure shows the response of first difference of the logarithm of the granular dispersion of the marginal product of capital. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 519 repetitions.

Figure 10: Effect of granular residual shock $\Theta_{100,t}^3$ on granular misallocation.



Notes: Responses of the level and first difference of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular residual shock. The left figure shows the response of level, and the right figure shows the response of first difference of the logarithm of the granular dispersion of the marginal product of capital. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 519 repetitions.

Figure 11: Effect of granular skewness shock Υ_{100}^3 on granular misallocation.



Notes: Responses of the level and first difference of the logarithm of the granular dispersion of the marginal product of capital to a negative one standard deviation granular skewness shock. The left figure shows the response of level, and the right figure shows the response of first difference of the logarithm of the granular dispersion of the marginal product of capital. The 95 percent confidence intervals of the empirical estimations are computed with a wild bootstrap of 519 repetitions.