Aggregate Consequences of Credit Subsidy Policies: Firm Dynamics and Misallocation*

In Hwan Jo† and Tatsuro Senga‡

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Abstract

Government policies that attempt to alleviate credit constraints faced by small and young firms are widely adopted across countries. We study the aggregate impact of such targeted credit subsidies in a heterogeneous firm model with collateral constraints and endogenous entry and exit. A defining feature of our model is a non-Gaussian process of firm-level productivity, which allows us to capture the skewed firm size distribution seen in the Business Dynamics Statistics (BDS). We compare the welfare and aggregate productivity implications of our non-Gaussian process to those of a standard AR(1) process. While credit subsidies resolve misallocation of resources and enhance aggregate productivity, increased factor prices, in equilibrium, reduce the number of firms in production, which in turn depresses aggregate productivity. We show that the latter indirect general equilibrium effects dominate the former direct productivity gains in a model with the standard AR(1) process, as compared to our non-Gaussian process, under which both welfare and aggregate productivity increase by subsidy policies.

Keywords: misallocation, collateral constraints, firm dynamics, firm size
JEL Classifications: E22, G32, O16

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†National University of Singapore, email: ecsji@nus.edu.sg, address: 1 Arts Link, Singapore 117570

‡Queen Mary University of London, RIETI and ESCoE, email: t.senga@qmul.ac.uk, address: Mile End Road, London, E1 4NS, UK

1 Introduction

Government policies that attempt to alleviate credit constraints faced by small and young firms are widely adopted across countries; provision of subsidized credit is a prime example of this.[1] Despite the popularity of such targeted industrial policies, quantitative studies on their macroeconomic effects are scarce.[2] Subsidized credit helps small and young firms achieve efficient and larger scales of production, thus resolving misallocation of resources and enhancing aggregate productivity. However, increased factor prices in equilibrium reduce the number of firms in production, which in turn depresses aggregate productivity. The relative magnitudes of each channel—the direct productivity gains and the indirect general equilibrium effects—depend on the underlying distribution of firms and their financial status. Thus, whether long-run effects are productivity enhancing or not is ambiguous.

In this paper, we offer a general equilibrium analysis of such targeted credit subsidy policies by extending a heterogeneous firm model with collateral constraints and endogenous entry and exit. In particular, we employ a Pareto-distributed firm productivity process to capture the skewed firm size distribution in the Business Dynamics Statistics (BDS). From our policy experiments, we have two main findings. First, aggregate productivity rises by at most 1.35 percent, following a credit subsidy policy for small or young firms. While the policy promotes the reallocation of resources among firms, associated indirect effects offset much of the potential gains in productivity. These indirect effects arise from adjustments in the extensive margins which reduce the equilibrium number of firms. Second, credit subsidy policies *decrease* aggregate productivity when a model is inconsistent with the empirical firm size distribution. This is the case when we instead use a standard AR(1) process of firm productivity, a common approach in the literature.[3] It follows that the skewness of the firm size distribution is crucial in quantitatively evaluating the aggregate effects of such policies.

Why do both extensive margin adjustments and firm size distribution matter? Models with a standard AR(1) process cannot capture firms located at the right tail of the skewed distribution of firms. Therefore, these models lack highly productive-small firms that

[1] For example, the US Small Business Administration (SBA) provides a loan guaranty for entrepreneurs starting or expanding small businesses under the 7(a) Loan Program. The volume of approved loans has been growing in recent years, reaching $24 bil. in 2015. We provide a summary of the program in Section 4.1.

[2] Guner, Ventura, and Xu (2008), Buera, Moll, and Shin (2013), and Buera, Kaboski, and Shin (2017) study the aggregate implications of policies aimed at specific groups of firms or entrepreneurs.

[3] See Hopenhayn and Rogerson (1993) for a similar approach. In business cycle studies, see Khan and Thomas (2008) and Bloom (2009) for the seminal works.

could grow substantially if collateral constraints are lifted, and for whom there are large potential gains from credit subsidy policies. In the absence of such firms with substantial growth potential, the direct effect from credit subsidy policies is relatively small and is dominated by the negative indirect effects in equilibrium. This indirect effect is amplified when we also match the left tail of the firm size distribution. In equilibrium, many small-unproductive firms endogenously respond to changes in the exit margin. When the extensive margins are held fixed, on the other hand, the number of firms remains constant by construction. A credit subsidizing policy then leads to a positive gain in aggregate productivity, even when the assumed firm productivity follows an AR(1) process.

Our model builds on a standard heterogeneous firm model. The model has three key ingredients. First, as discussed above, we employ a non-Gaussian process for firm-level productivity which follows a bounded Pareto distribution. It is well-known that the empirical distribution of firm employment is highly skewed. That is, small firms dominate the business population, while large firms account for the largest fraction of aggregate employment.[4] Our specification of firm productivity successfully replicates the empirical firm size distribution in the model economy. Second, we allow both endogenous entry and exit by firms.[5] This element is essential to reproducing the empirical patterns of firm dynamics, including substantially lower survival rates among young firms. In addition, it allows us to examine the role of extensive margin adjustments in affecting aggregate productivity. Third, forward-looking collateral constraints are added, in the spirit of Kiyotaki and Moore (1997). The presence of collateral constraints restricts investment decisions at the firm level, thereby hindering immediate firm growth upon entry.[6] This, together with the second ingredient, characterizes the firm lifecycle aspects with external financing and the corresponding firm age distribution.[7]

We use this model to study the aggregate implications of targeted credit subsidy policies in a general equilibrium environment. In the absence of policy interventions, we require the model to match the key macroeconomic aggregates, firm size and age distributions, and firm dynamics patterns in the US. We then introduce a policy into the model

[4] In 2012, the share of employment at large firms was 51.6 percent while the employment share of small firms was 16.7 percent in the United States (Statistics of U.S. Businesses, 2012).

[5] Khan and Thomas (2013), Buera and Moll (2015), and Catherine et al. (2018) abstract from endogenous exit margins. Buera, Kaboski, and Shin (2011, 2017) study the importance of endogenous extensive margins. Midrigan and Xu (2014) also explored this in their model.

[6] See Bahaj, Foulis, and Pinter (2017) for the evidence on collateral constraints and firm investment.

[7] There is extensive research on the importance of firm age. See, for example, Davis, Haltiwanger, and Schuh (1996), Dunne, Roberts, and Samuelson (1989), and Evans (1987). More recently, Fort et al. (2013) focused on the firm age dimension during the Great Recession. Hsieh and Klenow (2014) examined the lifecycle of plants and misallocation.

which provides credit to financially constrained firms economy-wide. In this regard, our approach is in line with that of Buera, Kaboski, and Shin (2017), who studied the macroeconomic impact of large-scale microfinance in developing economies and highlighted the importance of considering the general equilibrium effects of such policies. To avoid potential policy distortions, we assume that credit subsidies are lump-sum cash transfers from households to targeted firms.[8]

From our policy experiments, we show that there are four effects at work, one direct and three indirect, following a targeted credit subsidy. The direct effect emerges from the fact that small and young firms can achieve an efficient scale of production through subsidies. In the aggregate, the policy alleviates the capital misallocation that exists across firms due to credit constraints. In line with the findings in previous studies (e.g., Buera, Kaboski, and Shin (2011), Khan and Thomas (2013), Midrigan and Xu (2014)), we demonstrate the quantitative importance of misallocation in explaining aggregate productivity. In the meantime, the policy also brings indirect effects in general equilibrium. First, increased demand for capital and labor—by the recipients of the subsidy–raise factor prices. Higher factor prices depress the scale of production of untargeted firms. The second indirect effect is from adjustments in the extensive margins. Increased factor prices affect both entry and exit thresholds, together with the policy that encourages more entry by small firms. This is a *cleansing effect* that replaces less-productive incumbents with productive entrants.[9] The last effect is that there are fewer firms in operation because of higher costs of production, thus depresses aggregate productivity.[10]

Our policy experiments focus on both small and young firms. It is widely known that these firms are likely to experience more difficulties in financing their investment externally. For example, 61 percent of small firms reported that they faced financial challenges and 76 percent among them used personal funds, according to the Small Business Credit Survey 2016.[11] Young firms in the survey faced relatively tougher borrowing conditions due to a lack of credit history or limited access to credit. In addition, the recent literature has documented that credit constraints faced by young firms compound adverse shocks,

[8] See Catherine et al. (2018) for a similar application on the impact of an investment tax credit.

[9] We examine the cleansing effect of industrial policies, while the business cycle literature has investigated the cleansing effect of recessions. See Caballero and Hammour (1994) and Osotimehin and Pappada (2017), for example.

[10] Under the decreasing-returns-to-scale, the number of production units in a model is positively associated with the level of measured total factor productivity, holding aggregate factor inputs fixed. See Khan, Senga, and Thomas (2016) for a similar channel.

[11] The survey reports the business condition and the financial environment faced by small and young businesses in the US. 10,303 responses were collected by 12 regional Federal Reserve Banks.

which leads to high exit rates among them.[12] Given its pervasiveness, we focus on a subsidized credit policy that alleviates difficulties in external borrowing for small and young firms, as in Buera, Moll, and Shin (2013).

In practice, there are a variety of government support schemes for small businesses.[13] These actual policies are mostly in comprehensive packages which include legal and financial assistance, education of managerial skills, and so forth. Moreover, such policies are in small-scale for a limited number of eligible firms. In contrast, we attempt to isolate the macroeconomic impact of policies that are specialized to ease credit constraints among firms. One such policy is the SBA loan program in the US, noted above.[14] We also provide a plausible and useful model counterpart of this policy in this paper. The corresponding outcome from the model is consistent with our main findings though the magnitudes are smaller, and we verify that the indirect effects described above are robust.

Across our policy experiment results, there are two main mechanisms affecting the allocative efficiency of resources, one across incumbents (e.g., Restuccia and Rogerson (2008)), and one across entering and exiting firms.[15] The role of financial frictions in generating resource misallocation and its aggregate implications have been studied by Khan and Thomas (2013), Buera and Moll (2015), Buera, Kaboski, and Shin (2011), and Midrigan and Xu (2014), among others. More recently, Catherine et al. (2018) studied misallocation in a model of collateral constraints and found a positive productivity gain from investment subsidies. Our policy exercises are distinct as we allow endogenous firm entry and exit, which leads to our finding that the long-run effect on aggregate productivity from alleviating credit constraints may be negative.

Our focus here is on the role of policies in reducing micro-level distortions. Previous studies, in contrast, have looked at policies that distort the allocation of resources. Guner, Ventura, and Xu (2008) considered a model with size-contingent regulations and used their calibrated version of the model to show sizable welfare losses. Gourio and Roys

[12] Collier et al. (2016) discuss the role of credit constraints in managing negative shocks.

[13] Beginning in January 2011, the Obama Administration promoted a set of entrepreneur-focused policies, including the Jumpstart Our Business Startups (JOBS) Act. The White House (2012) claims that these policies allow *"Main Street small businesses and high-growth enterprises to raise capital from investors more efficiently, allowing small and young firms across the country to grow and hire faster."* The targets of the policy initiatives include finance, education, red tape, innovation, and market opportunities for entrepreneurs.

[14] Similar but different industrial policies have been implemented in European and Asian countries. A prime example includes the Moratorium Act in Japan enacted in 2009. This allows small businesses to defer their debt repayment backed by government guaranties, while requiring banks to *"endeavor to take steps to alleviate the burden of debt."* The total amount of rescheduled loans is about $8 tn. in 2012.

[15] Seminal empirical works on the allocative efficiency include Hsieh and Klenow (2009) and Bartelsman, Haltiwanger, and Scarpetta (2013).

(2014) also studied the aggregate implication of a French policy that distorts the size distribution of firms, showing the important role of such policy distortion in explaining aggregate productivity.[16]

We present our theory in Section 2. We calibrate the theoretical model and examine the equilibrium before policy intervention in Section 3. In Section 4, we analyze the impact of industrial policies, and Section 5 investigates the role of extensive margins and skewed firm size distribution. Section 6 concludes the paper.

2 Model

Time is discrete in infinite horizon. There are a large number of heterogeneous firms producing a homogeneous good. Firms are subject to persistent shocks to individual productivity and face collateral constraints. These, together with endogenous entry and exit, yield substantial heterogeneity in production. Households are identical and infinitely-lived. We abstract from aggregate uncertainty and consider a stationary industrial equilibrium.

In the following sub-sections, we present our model economy and examine the optimization problem of firms, followed by the household problem and the definition of recursive competitive equilibrium. We then characterize firm-level decisions in a tractable way to render our counterfactual exercises feasible. In particular, we summarize two firm-level states, capital and borrowing, by using a single-state variable defined as cash-on-hand.

2.1 Firms

2.1.1 Production and Financial Friction

The economy consists of a continuum of firms. Each firm owns its predetermined capital stock, k, and hires labor, n. The production technology is described by $y = \epsilon F(k, n)$, where $F(\cdot)$ exhibits decreasing-returns-to-scale (DRS) and ϵ represents firm-level productivity. We assume that $\epsilon \in \mathbf{E} \equiv \{\epsilon_1, \epsilon_2, \ldots, \epsilon_{N_\epsilon}\}$ follows a Markov chain with $\pi^\epsilon_{ij} \equiv Pr(\epsilon' = \epsilon_j | \epsilon = \epsilon_i) \geq 0$ and $\sum_{j=1}^{N_\epsilon} \pi^\epsilon_{ij} = 1$ for each $i = 1, 2, \ldots, N_\epsilon$, where primes denote future values for notational convenience.

Not all firms are able to finance their desired investment due to financial frictions. All debt is priced at q, and each firm faces a borrowing limit on this one-period discount

[16] See Garicano, Lelarge, and Van Reenen (2016) and Braguinsky, Branstetter, and Regateiro (2011) for econometric analyses of size-dependent policies.

debt. This borrowing constraint restricts the amount of new debt level, b', not to exceed a firm's collateral. Based on the idea of limited enforceability of financial contracts, we assume that the firm's future period capital, k', serves as collateral for current borrowing. Therefore, for a firm choosing k' in the current period, the collateral constraint is given by $b' \leq \theta k'$, where the financial parameter, θ, captures financial frictions at the economy-wide level. Notice that θ is assumed to be common across firms, but the borrowing decision depends endogenously on each firm's state. When θ is close to the real interest rate, $\frac{1}{q}$, the financial market allows firms to invest at their desired level. Note that our specification of collateral constraints is forward-looking in the spirit of Kiyotaki and Moore (1997), while we abstract from their feedback channel of asset prices.

In the presence of collateral constraints, firms gradually accumulate capital. Once they achieve a capital level consistent with their expected productivity, they can also accumulate financial savings, $b' < 0$. This implies that a borrowing constraint is binding for some but not all firms in a given period. In this way, the model reproduces the observed heterogeneity in the reliance on external borrowing.

2.1.2 Entry and Exit

We model firm entry and exit based on the standard approaches in the literature.[17] In each period, incumbent firms may exit the economy either by an exogenous probability or by their endogenous decisions. Due to the financial frictions at the firm-level, individual states of productivity, capital, and borrowing jointly affect the latter endogenous exit decision. Together with endogenous entry decisions by potential entrants, our model features realistic firm dynamics.

At the beginning of each period, firms are informed of their respective status of exit which takes place after production, and the fixed probability of exit, $\pi_d \in (0, 1)$, is common across firms.[18] The remaining firms after this exogenous exit need to pay ξ_o units of output in order to continue operation in the next period. This fixed cost of operation creates a binary exit decision. If a firm does not pay this cost, it has to exit the economy permanently which implies its value is zero. Thus, only the firms continuing to the next period make intertemporal decisions about investment and borrowing after paying

[17] Hopenhayn (1992) is the seminal work in this literature with industry dynamics driven by firms' endogenous entry and exit. Clementi and Palazzo (2016) modify the timing of entry in the Hopenhayn model to investigate the business cycle implications of firm dynamics. We follow the similar approach of Clementi and Palazzo, while introducing the exogenous exit as employed in Khan and Thomas (2013).

[18] This is a simple way to avoid the Modigliani-Miller environment where financial frictions are irrelevant when firms survive long enough. Further, the exogenous exit assumption helps the model reproduce the empirical distribution of firm age by allowing turnovers of large-mature firms.

ξ_o.[19] This endogenous exit margin of firm dynamics enables relatively less-profitable firms to endogenously choose to exit, which can potentially reduce the degree of resource misallocation arising from financial frictions in the absence of policy interventions.

We further assume that there is a fixed measure, M^e, of potential entrants in each period. The potential entrants are uniformly distributed over their initial capital and debt combination, (k_0, b_0), and the initial productivity of a potential entrant, ϵ_0, is randomly drawn from the ergodic distribution of $\epsilon \in \mathbf{E}$.[20] When a potential entrant decides to enter, it needs to pay a fixed entry cost, ξ_e, in units of output. In this way, we are able to set up a simple binary problem of endogenous entry. Note that firm entry takes place at the end of a period, and actual entrants start operating in the next period, given their initial state, (k_0, b_0, ϵ_0).

2.1.3 Timing and Firm Distribution

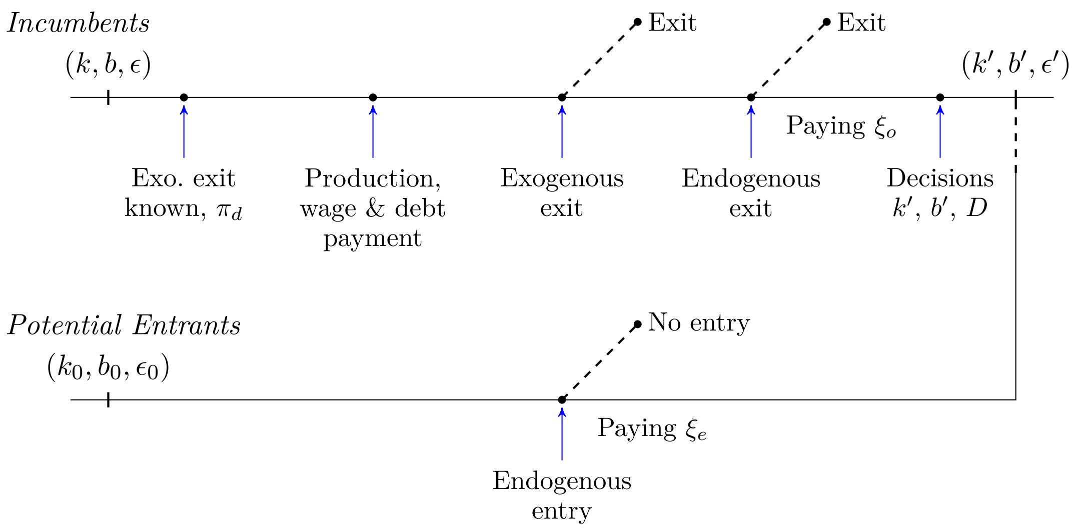
At the beginning of a period, an incumbent firm is identified by its individual state vector, (k, b, ϵ); the predetermined capital, $k \in \mathbf{K} \subset \mathbb{R}_+$; the amount of debt carried from the previous period, $b \in \mathbf{B} \subset \mathbb{R}$; and the current period idiosyncratic productivity level, $\epsilon \in \mathbf{E}$. We summarize the distribution of firms by a probability measure, $\mu(k, b, \epsilon)$, which is defined on a Borel algebra $\mathcal{S} \equiv \mathbf{K} \times \mathbf{B} \times \mathbf{E}$.

We illustrate the timing of the model in a given period as shown in the following diagram. Given the current state, (k, b, ϵ), an incumbent firm maximizes the sum of its current and future expected discounted dividends, considering its possible exit within each period. At the beginning of a period, the firm realizes its exogenous exit status after production. Once production is completed, all firms make payments for wage bill and existing debt, and then π_d share of firms on $\mu(k, b, \epsilon)$ disappears exogenously. The remaining firms decide whether to continue to the next period by paying the fixed operation cost, ξ_o. Conditional on staying, firms undertake intertemporal decisions on investment, i, and borrowing, b', while determining their current dividend payments, D. The capital accumulation of each firm is standard, $i = k' - (1 - \delta)k$, with $\delta \in (0, 1)$.

[19] Both exogenous and endogenous exit occur after production is completed in a given period. This ensures that all existing debt is repaid by the exiting firms.

[20] The distribution of potential entrants is not observable in the data, so we choose to assume a uniform distribution. k_0 and b_0 are jointly drawn from a density, $\int_0^{\bar{k}_0} \int_0^{\bar{b}_0} (\bar{k}_0 \cdot \bar{b}_0)^{-1} (d[k_0 \times b_0])$. We assume that $\bar{k}_0$ corresponds to a fraction, χ_e, of the capital choice without financial frictions at the medium productivity value of ϵ, and set $\bar{b}_0 = \theta_e \bar{k}_0$, where θ_e is the maximum leverage of potential entrants.

Timing within a Period



Meantime, potential entrants draw their initial state, (k_0, b_0, ϵ_0), and decide whether to enter the economy by paying the fixed entry cost, ξ_e.[21] Given its initial state, an entering firm starts operating in the next period, along with the continuing incumbents in the above. Markets are perfectly competitive, so firms take the wage rate, w, and the discount debt price, q, as given.

2.1.4 Firm's Problem

Given $\epsilon_i \in \mathbf{E}$, let $v^0(k, b, \epsilon_i)$ be the value of a firm at the beginning of the current period, before its survival from exogenous exit is known. Accordingly, define $v^1(k, b, \epsilon_i)$ as a surviving firm's value, before making its decision to pay the operation cost ξ_o. Finally, if the firm decides to continue to the next period, its value is given by $v(k, b, \epsilon_i)$. Then the firm's optimization problem can be recursively defined by using v^0, v^1, and v.

$$\begin{aligned} v^0(k, b, \epsilon_i) = \pi_d \cdot \max_n \left[\epsilon_i F(k, n) - wn + (1-\delta)k - b\right] \\ + (1-\pi_d) \cdot v^1(k, b, \epsilon_i) \end{aligned} \tag{1}$$

[21] In general, the selection effect at the entry margin may differ by assumed timing. In Hopenhayn (1992), there is a small selection effect since only firms with high productivity enter. As discussed by Clementi and Palazzo (2016), the entry timing such as ours leads to richer heterogeneity across entrants and thus stronger selection effect. We thank an anonymous referee for pointing this out.

$$v^1(k, b, \epsilon_i) = \max\left\{0, -\xi_o + v(k, b, \epsilon_i)\right\} \tag{2}$$

In Equation (1), the firm takes the possibility of exogenous exit into account. In case it is destined to exit by π_d, the firm maximizes its liquidation value at the end of the period without dynamic decisions. In case of surviving, on the other hand, the firm makes a binary decision over the value of zero, *(endogenous exit)*, and the value of $-\xi_o + v(k, b, \epsilon_i)$, *(stay)*, in Equation (2). The value of continuing to the next period is given by $v(k, b, \epsilon_i)$ as below.

$$v(k, b, \epsilon_i) = \max_{n,k',b',D} \left[D + \beta \sum_{j=1}^{N_\epsilon} \pi_{ij}^{\epsilon} v^0(k', b', \epsilon_j)\right] \tag{3}$$

$$\begin{aligned} &\text{subject to} \\ &0 \leq D \equiv \epsilon_i F(k, n) - wn + (1-\delta)k - b - k' + qb' \\ &b' \leq \theta k' \end{aligned}$$

In (3), the firm optimally chooses its labor demand, n, future capital, k', and new debt level, b', to maximize the sum of the firm's current dividends, D, and the beginning-of-the-period value, $v^0(k', b', \epsilon_j)$, in the next period. Current period dividends are the residual defined in the firm's budget constraint, and we restrict them to be non-negative. Firms discount their future expected values using $\beta \in (0, 1)$.[22] Financial frictions are introduced in a collateral constraint on the firm's new borrowing in the above maximization problem.

Lastly, we define the value of a potential entrant with (k_0, b_0, ϵ_0) as v^e.

$$v^e(k_0, b_0, \epsilon_0) = \max\left\{0, -\xi_e + \beta v^0(k_0, b_0, \epsilon_0)\right\} \tag{4}$$

The potential entrant makes a binary decision to pay or not to pay the entry cost ξ_e. Once it enters, the firm starts operation in the next period given its initial state, as indicated in the value of entry in (4).

[22] We also use β to denote households' subjective discount factor in the next sub-section. Abstracting from aggregate uncertainty, there is no stochastic discount factor in the above firm's problem. Thus, the equilibrium discount debt price, q, is equal to β.

2.2 Households and Equilibrium

2.2.1 Representative Household

We assume that there is a unit measure of identical households in the economy. In each period, households earn their labor income by supplying a fraction of their time endowment. Period utility is given by $U(C, 1-N)$, and households discount future utility by a subjective discount factor, β. The representative household holds a comprehensive portfolio of assets; firm shares of measure λ and non-contingent discount bonds ϕ. It maximizes lifetime expected discounted utility by choosing the quantities of aggregate consumption demand, C^h, and labor supply, N^h, while adjusting its asset portfolio. The household value, V^h, is defined as below.

$$\begin{aligned} V^h(\lambda, \phi) = & \max_{C^h, N^h, \lambda', \phi'} \left[U(C^h, 1-N^h) + \beta V^h(\lambda', \phi') \right] \qquad (5) \\ & \text{subject to} \\ & C^h + q\phi' + \int_{\mathcal{S}} \rho_1(k', b', \epsilon')\lambda'(d[k \times b \times \epsilon]) \\ & \quad \leq wN^h + \phi + \int_{\mathcal{S}} \rho_0(k, b, \epsilon)\lambda(d[k \times b \times \epsilon]) \end{aligned}$$

We apply the following notation for stock prices. In (5), $\rho_1(k', b', \epsilon')$ denotes the ex-dividend prices of firm shares in the current period, and $\rho_0(k, b, \epsilon)$ is the dividend-inclusive value for current shareholding, λ. Let $\Phi^h(\lambda, \phi)$ be the household's decision for bonds and $\Lambda^h(k', b', \epsilon', \lambda, \phi)$ its choice of firm shares corresponding to the future state (k', b', ϵ').

2.2.2 Recursive Competitive Equilibrium

We consider a stationary industrial equilibrium of the model, where the distribution of firms, $\mu(k, b, \epsilon)$, is time-invariant. In the following, we define recursive competitive equilibrium. For simplicity, we denote the distribution of producing firms as μ^p, the measure of actual entrants as μ^e, and the measure of endogenously exiting firms as μ^{ex}. Whenever possible, we suppress the arguments of functions in the definition below.

A stationary *recursive competitive equilibrium* is a set of functions including prices $\left(w, q, \rho_0, \rho_1\right)$, quantities $\left(N, K, B, D, C^h, N^h, \Phi^h, \Lambda^h\right)$, a distribution $\mu(k, b, \epsilon)$, and values $\left(v^0, v^1, v, v^e, V^h\right)$ that solve the optimization problems and clear markets in the following conditions.

1. v^0, v^1, and v solve Equations (1)–(3), and (N, K, B, D) are the associated policy functions for firms.
2. V^h solves (5), and $(C^h, N^h, \Phi^h, \Lambda^h)$ are the associated policy functions for households.
3. The labor market clears, $N^h = \int_{\mathcal{S}} N(k, \epsilon) \cdot \mu^p(d[k \times b \times \epsilon])$.
4. The goods market clears.

$$
\begin{aligned}
C^h = &\left(\int_{\mathcal{S}} \Big[\epsilon F(k, N) - (1-\pi_d)\big(K(k, b, \epsilon) - (1-\delta)k\big) + \pi_d(1-\delta)k - \xi_o \Big] \right. \\
&\left. \cdot \mu^p(d[k \times b \times \epsilon]) \right) + \int_{U(0, \bar{k}_0)} (k_0 - \xi_e) \cdot \mu^e(d[k_0 \times b_0]) \\
&- \int_{\mathcal{S}} k \cdot \mu^{ex}(d[k \times b \times \epsilon])
\end{aligned}
$$

5. The distribution of firms, $\mu(k, b, \epsilon)$, is a fixed point where its transition is consistent with the policy functions, (K, B), and the law of motion for ϵ.

2.3 Characterizing Firm-Level Decisions

2.3.1 Firm Types

To characterize the firm-level decision rules in the model, it is convenient to first define a subset of firms in the distribution whose decisions are not affected by the collateral constraints in any possible future state. In particular, we follow the approach of Khan and Thomas (2013) to distinguish firm types in our model economy. Note that this distinction is solely for deriving all the intertemporal decisions made by firms, and that a firm may alter its type depending on the state variables.

We define a firm as *unconstrained* when it has already accumulated sufficient wealth such that it never experiences binding borrowing constraints in any possible future state. In this case, all the unconstrained firm's Lagrangian multipliers on its borrowing constraints become zero, and the firm is indifferent between dividend payments and retained earnings. On the other hand, the remaining firms in the distribution are defined as *constrained*. Constrained firms may or may not experience binding borrowing constraints in the current period; thus, they choose to pay zero dividends in the current period as the shadow value of retained earnings is greater than that of paying dividends. In the following, we begin with deriving the decision rules by unconstrained firms in the model.

First, we derive the optimal static labor choice, which applies to all firms. Labor choices are frictionless and a firm with (k, ϵ) chooses $n = N^w(k, \epsilon)$, which solves the static labor condition, $\epsilon D_2 F(k, n) = w$.

Next, we derive the choice of future capital, k', by the unconstrained firms. The collateral constraint is irrelevant for this type of firms, so we can easily derive their optimal level of $k' = K^w(\epsilon)$ as follows. Let $\Pi^w(k,\epsilon) \equiv \epsilon F(k, N^w) - wN^w$ be the current earnings of a firm with its optimal labor hiring N^w. Given the Markov property for the assumed stochastic processes and the absence of capital adjustment costs, K^w is the solution to the following problem.

$$\max_{k'} \Big[-k' + \beta \sum_{j=1}^{N_\epsilon} \pi_{ij}^{\epsilon} \big(\Pi^w(k', \epsilon_j) + (1-\delta)k' \big) \Big]$$

With the policy functions N^w and K^w, the *minimum savings policy*, $b' = B^w(\epsilon)$, is recursively defined by the following two equations.

$$B^w(\epsilon_i) = \min_{(\epsilon_j)_{j=1}^{N_\epsilon}} \Big(\tilde{B}\big(K^w(\epsilon_i), \epsilon_j\big) \Big) \tag{6}$$

$$\begin{aligned} \tilde{B}(k, \epsilon_i) = {} & \epsilon_i F(k, N^w) - wN^w + (1-\delta)k - K^w(\epsilon_i) \\ & + q \min \big\{ B^w(\epsilon_i), \theta K^w(\epsilon_i) \big\} \end{aligned} \tag{7}$$

$\tilde{B}(K^w, \epsilon_j)$ in (6) denotes the maximum level of debt (or the minimum level of saving) that an unconstrained firm can hold at the beginning of the next period in which $\epsilon' = \epsilon_j$ is realized. Having chosen the unconstrained choice of capital, K^w, at the current period, the firm will remain unconstrained in the subsequent periods by definition. The minimum savings policy, B^w, ensures that the firm's debt never exceeds this threshold level, $\tilde{B}$, given all possible realizations of ϵ. Moreover, the threshold function can be retrieved by using B^w and K^w at the current period state with (k, ϵ_i), as in (7). Notice that the minimum operator in (7) reflects the collateral constraint in the firm's problem. In (6), B^w again is determined to have the unconstrained firms unaffected by the constraint over any future path of ϵ.

2.3.2 Cash-on-Hand and Decision Rules

The incumbent firm's problem in (3) is a challenging object because of the occasionally binding constraints for D and b'. In addition, notice that a firm's individual state vector includes two endogenous variables, (k, b), with each having a continuous support. However, levels of k and b of firms do not separately determine the choices of k' and b'. This is due to the absence of any real adjustment costs in the model, and therefore we can collapse these two continuous individual state variables into a newly defined variable called *cash-on-hand*. We define the cash-on-hand, $m(k,b,\epsilon)$, of a firm with (k,b,ϵ) by using the optimal labor demand N^w.

$$m(k,b,\epsilon) \equiv \epsilon F(k,N^w) - wN^w + (1-\delta)k - b$$

Notice that the decisions of k' and b' made by continuing incumbents determine the level of cash-on-hand in the future period, $m(k',b',\epsilon')$, along with the realization of ϵ'. By using this new state variable, we rewrite the incumbent firm's problem in (1)–(3) using the following values W^0, W^1, and W.

$$W^0(m,\epsilon_i) = \pi_d \cdot m + (1-\pi_d)\cdot W^1(m,\epsilon_i) \tag{8}$$

$$W^1(m,\epsilon_i) = \max\big\{0, -\xi_o + W(m,\epsilon_i)\big\} \tag{9}$$

$$W(m,\epsilon_i) = \max_{k',b',D,m'_j}\Big[D + \beta\sum_{j=1}^{N_\epsilon}\pi^\epsilon_{ij}W^0(m'_j,\epsilon_j)\Big] \tag{10}$$

$$\begin{aligned}
&\text{subject to}\\
&0 \le D \equiv m - k' + qb'\\
&b' \le \theta k'\\
&m'_j \equiv m(k',b',\epsilon_j)\\
&\quad = \epsilon_j F\big(k', N^w(k',\epsilon_j)\big) - wN^w(k',\epsilon_j) + (1-\delta)k' - b'
\end{aligned}$$

Having solved the unconstrained policies K^w and B^w, we define a threshold level of m for each ϵ that distinguishes the unconstrained firms. From the firm's budget constraint in (10), an unconstrained firm pays the current dividends, $D^w = m - K^w + qB^w \ge 0$. It follows that this firm's cash-on-hand is greater than or equal to a certain threshold level, $\tilde{m}(\epsilon) \equiv K^w(\epsilon) - qB^w(\epsilon)$. Therefore, any firm with $m(k,b,\epsilon) < \tilde{m}(\epsilon)$ can be identified as constrained.

Recall that, in any given period, some constrained firms experience currently binding borrowing constraints while the others do not. We call the latter constrained firms *Type-1*, and the firms with currently binding constraints *Type-2*. Notice that Type-1 firms can still adopt the unconstrained capital policy, K^w, but not the minimum savings policy, B^w. The debt policy of Type-1 firms can be easily determined by the zero-dividend policy after substituting $k' = K^w$ in the budget constraint. On the other hand, Type-2 firms can only invest to the extent allowed by their borrowing limits. This constrained choice of capital is derived from the level of cash-on-hand, m, held by Type-2 firms. Specifically, $D = 0$ implies that a Type-2 firm's decision of k' and b' must be feasible, given the firm's available cash-on-hand. Given m, we define the upper bound of a Type-2 firm's capital choice as $\bar{K}(m) \equiv \frac{m}{1-q\theta}$, from the binding collateral constraint. Firms with more cash-on-hand, therefore, can relax this upper bound until they can choose the unconstrained capital policy, K^w. Notice also that the upper bound, $\bar{K}(m)$, approaches infinity as the financial parameter θ becomes closer to q^{-1}, which illustrates the case of perfect credit markets. Below, we summarize the decision rules of k' and b' by firm type, given (m, ϵ).

- Firms with $m \geq \tilde{m}(\epsilon)$ are *unconstrained* and therefore adopt $K^w(\epsilon)$ and $B^w(\epsilon)$.
- For *constrained* firms with $m < \tilde{m}(\epsilon)$, the upper bound for k' is $\bar{K}(m) \equiv \frac{m}{1-q\theta}$.
  - Firms with $K^w(\epsilon) \leq \bar{K}(m)$ are *Type-1* and adopt $k' = K^w(\epsilon)$ and $b' = \frac{1}{q}\big(K^w(\epsilon) - m\big)$.
  - Firms with $K^w(\epsilon) > \bar{K}(m)$ are *Type-2* and adopt $k' = \bar{K}(m)$ and $b' = \frac{1}{q}\big(\bar{K}(m) - m\big)$.

3 Model Parameters and Steady State

We numerically solve the model by using non-linear methods, and find a stationary equilibrium where individual decisions are consistent with market clearing prices. In Section 3.1, we calibrate the model to be consistent with the observed data in the US. Once the model is calibrated, in Section 3.2, we show how a firm's individual state is related to its decision of investment, borrowing, entry and exit. This helps us understand the role of key model ingredients, in the absence of any policy intervention. Based on this initial equilibrium, we consider policy counterfactual experiments in the following section.

3.1 Calibration

The model is annual, and we assume the standard functional forms of household preference and production technology in the literature. The period utility function of the representative household features the indivisible labor, $U(C, 1-N) = \log C + \psi(1-N)$, following Rogerson (1988).[23] The DRS production function is in Cobb-Douglas form, $F(k, n) = k^\alpha n^\nu$, with $\alpha, \nu > 0$ and $\alpha + \nu < 1$. Given the assumed functional forms, we set the model parameter values to jointly match the key macroeconomic moments and the firm-level heterogeneity observed in the US data. Table 1 reports the parameter values used in our calibration, and Table 2 summarizes the resulting moments from the model along with the corresponding US data.

[23] The assumed preference is used in Hopenhayn and Rogerson (1993), to study the macroeconomic implications of firing costs. In the appendix, we consider a case with inelastic labor supply, and our main results are robust.

Table 1 : Parameter Values

Model Parameter Values			
α	0.280	θ	0.820
β	0.960	M^e	0.300
δ	0.069	χ_e	0.0436
ν	0.600	N_ϵ	13
ψ	2.480	ρ_ϵ	0.750
π_d	0.100	ϵ_m	0.393
ξ_o	0.03392	ϵ_M	1.122
ξ_e	0.00991	γ_ϵ	4.420

First, we set the value of the subjective discount factor, β, to imply the long-run real interest rate of about 4 percent per annum. The curvature of labor input in the production function, ν, is set to have the average share of labor income of 0.6, following Cooley and Prescott (1995). We set the preference parameter of labor disutility, ψ, to get the average hours worked of 0.33 at the stationary equilibrium of the model. The annual depreciation rate, δ, is chosen to match the average aggregate investment to capital ratio of the postwar US economy, while the production parameter, α, is set to be consistent with the average capital to output ratio of 2.3. For the aggregate time series of investment, output, and private capital, we use the Fixed Asset Tables and National Income and Product Accounts (NIPA) from the Bureau of Economic Analysis (BEA) between 1954 and 2007. The parameter value in the collateral constraint, θ, is 0.82, to imply the average debt to capital ratio of the non-farm, non-financial businesses in the Flow of Funds from 1954 to 2007.

We set the exogenous exit rate in the model, π_d, at 0.10 and the fixed measure of potential entrants, M^e, at 0.30. We further assume that the maximum leverage of potential entrants, θ_e, is the half of that of incumbents, θ. The rest of model parameter values are calibrated to reproduce the observed size and age distributions of firms and the moments of firm entry and exit in the data. The parameter values of the fixed operation cost, ξ_o, and the entry cost, ξ_e, largely determine the quantitative magnitude of the entry and exit margins, given a value of χ_e which determines the relative size of the largest entrants, $\bar{k}_0$, to that of incumbents. Our calibrated model implies both the average total exit rate of private firms (11 percent) and the relative total employment by entrants to the aggregate (3 percent) are close to the BDS data. In addition, the above parameters governing firm dynamics in the model critically affect the resulting shape of firm age distribution. Table 2 shows that our model also successfully matches the average distribution of firm age in the BDS.

Table 2 : Moments

Aggregate Moments		
Description	Data	Model
Average hours worked	-	0.333
Measure of firms, $\int \mu \cdot d\mu$	-	1.000
Investment to capital ratio (BEA)	0.069	0.069
Capital to output ratio (BEA)	2.300	2.222
Debt to capital ratio (Flow of Funds)	0.567	0.564
Total exit rate (BDS)	0.110	0.114
Employment share of entrants (BDS)	0.033	0.033

Firm Size Distribution					**Firm Age Distribution**		
	Employment Share		Population Share			Population Share	
Employees	Data	Model	Data	Model	Age	Data	Model
1 to 4	0.0584	0.0584	0.5506	0.5412	0	0.1236	0.1176
5 to 19	0.1455	0.1455	0.3342	0.3299	1	0.0950	0.0870
20 to 99	0.1814	0.1814	0.0964	0.0911	2	0.0806	0.0766
100 to 499	0.1395	0.1395	0.0153	0.0225	3	0.0697	0.0691
500 to 2499	0.1179	0.1179	0.0026	0.0083	4	0.0613	0.0624
2500+	0.3573	0.3573	0.0009	0.0071	5	0.0548	0.0564
					6 to 10	0.1939	0.2104
SME (1-499)	0.5248	0.5248	0.9965	0.9847	11 to 15	0.1302	0.1271
					16+	0.1910	0.1934
					Young (0-4)	0.4302	0.4127

Note: The empirical firm size and age distributions are calculated from the annual tables in the BDS database. We report the average values from 1977 to 2007 for the size distribution, and from 1993 to 2007 for the age distribution, respectively. SME denotes small-medium sized enterprises.

In order to replicate the empirical firm size distribution, we assume that the idiosyncratic productivity, ϵ, is drawn from a time-invariant distribution, $G(\epsilon; \epsilon_m, \epsilon_M, \gamma_\epsilon)$, which is a bounded Pareto distribution. In each period, a firm in our model economy retains its previous level of individual productivity with a fixed probability, ρ_ϵ. We set $\rho_\epsilon = 0.75$ to be consistent with the evidence on the persistence of firm-level productivity in the data.[24] The bounds of ϵ support, (ϵ_m, ϵ_M), and the shape parameter, γ_ϵ, of the bounded Pareto distribution are chosen to have both the employment share and the population share in each firm size bin aligned with the corresponding average values reported in the BDS from 1977 to 2007. We discretize ϵ using 13 values in our numerical applications.[25]

[24] The constant hazard of resetting productivity is recently employed in the models of production heterogeneity. See Buera, Kaboski, and Shin (2011) or Buera and Shin (2013), for example. The average persistence of ϵ in a long simulation is very close to the value of ρ_ϵ. This falls into the range of the persistence estimates in Foster, Haltiwanger, and Syverson (2008).

[25] Our simulation of the assumed ϵ process results in the unconditional mean of 0.466 and the standard deviation of 0.113. The share of firms with productivity less than the mean value is about 73 percent.

Table 2 reports the model-generated firm size distribution and its empirical counterpart, in which we divide the employment size groups by 6 bins. As is well known in the literature, Table 2 shows that the empirical distribution of firm size is highly skewed, where more than 88 percent of firms hire fewer than 20 employees in a given year. We almost perfectly replicate this lower tail of the empirical firm size distribution in our model economy by employing the Pareto-distributed ϵ. By the nature of the collateral constraint that we assumed, moreover, small firms in the size distribution are more likely to be financially constrained when their productivity is relatively high whereas their cash-on-hand is insufficient.

Before we discuss the quantitative results from the model, we elaborate our procedure of computing the model-generated firm size distribution which is directly taken from Jo (2017). Given the stationary distribution of firms, $\mu(k, b, \epsilon)$, in equilibrium, we begin with constructing a cumulative distribution of employment by using $N^w(k, \epsilon)$. Based on the employment shares across size bins in the BDS, we find the employment threshold, $\bar{n}$, in each firm size group along the above cumulative distribution from the model. We then compute the measure of firms specifically located on each firm size bin which is defined from those employment thresholds. In sum, we first align the model employment shares by firm size to be exactly the same with the corresponding values in the BDS, and then choose the parameter values to generate the model population shares as closely as possible to the data.

3.2 Steady State: Firm Heterogeneity and Decisions

We begin with describing the stationary distribution of firms in the model. Figure 1 shows the entire distribution of cash-on-hand, $m(k, b, \epsilon)$, at the steady state. The distribution of m is highly skewed with more than 88 percent of firms holding cash-on-hand of less than 1. This represents the corresponding shape of the underlying firm size distribution in the model, as reported in Table 2 and Figure 2. It follows that our model generates substantial heterogeneity at the firm level that is endogenously determined by the combination of persistent productivity shocks, collateral constraints, and endogenous margins of firm entry and exit.

More importantly, firms with small cash-on-hand are those most likely to be financially constrained subject to their productivity and borrowing limits, as we discussed in Section 2. To see this more clearly, Figure 3 provides a snapshot of the decision rules of incumbent firms on capital, k', debt, b', and dividends, D, as functions of $m(k, b, \epsilon)$ at a specific value of ϵ. In the figure, we add the two vertical lines to distinguish the firm types by

$m(k, b, \epsilon)$: unconstrained , Type-1, and Type-2. The vertical line near $m = 100$ represents the threshold for being unconstrained, $\tilde{m}(\epsilon)$, and the one near $m = 20$ is the threshold for being Type-1 firms. Starting from the right-hand side of the figure, when a firm has survived and accumulated sufficient wealth over time such that $m \geq \tilde{m}(\epsilon)$, it is considered unconstrained. The firm then adopts the unconstrained choices of capital, $K^w(\epsilon)$, and debt, $B^w(\epsilon)$, and starts paying positive dividends. Constrained firms with m less than the above threshold value, in contrast, follow the zero-dividend policy to accumulate their internal savings in order to become unconstrained. Type-1 firms between the two thresholds can still adopt the optimal level of capital, $K^w(\epsilon)$, while gradually reducing debt as their m increases. Lastly, Type-2 firms with small m are only able to invest up to their borrowing limits, so their choice of capital is constrained with positive borrowing. From Figure 1, we observe that these Type-2 firms are concentrated at the lower tail of the cash-on-hand distribution while maintaining positive leverages.

In our model, young firms start relatively small upon entry and then gradually accumulate cash-on-hand over time. These lifecycle dynamics of incumbent firms are illustrated in Figure 4. At age 0, an average firm in our model economy is financially constrained because it is short on collateral for external financing. Thus, the firm keeps raising its external debt level until age 4 and then gradually de-leverages once it can finance the optimal level of investment for $K^w(\epsilon)$, around age 6. In addition, conditional on remaining in the economy, firms still accumulate financial savings even after age 18, so they eventually become unconstrained and pay positive dividends. This is represented by the hump-shaped leverage curve in the lower panel of the figure. Any policy targeting firms in a specific age group will therefore shift the average lifecycle dynamics in Figure 4, which eventually reshapes the entire firm distribution in the long-run.

Next, we look at the endogenous exit decision by incumbent firms at the steady state of the model. Figure 5 shows the exit choice (=1) over capital and debt over the ergodic distribution of ϵ. Consistent with conventional knowledge, a firm in the model decides to exit when it accumulates relatively larger debt than its existing capital stock. This is because the continuation value of such an over-leveraged firm falls below 0, and the firm thus finds it better to exit before paying the fixed operation cost as illustrated in Equation (4). Such vulnerable incumbents largely correspond to Type-2 firms in the model, and, in Figure 5, the exit decision occurs at the margin of positive leverages ($b/k > 0$), which is the case of binding collateral constraints. Young firms with both low productivity and small wealth are easily tempted to this outside option of exit, so we are also able to capture the disproportionately high exit rate among those firms. This endogenous margin of firm dynamics in our model naturally entails a cleansing mechanism that drives unprofitable firms out of the economy, which is in contrast to the case with only exogenous exit.

Lastly, Figure 6 presents the entry decision by potential entrants. Recall that the potential entrants are uniformly distributed over (k_0, b_0) at a given level of initial productivity, ϵ_0, which is drawn from a bounded Pareto distribution. We calculate the average decision of entry weighted over ϵ_0, and the brightest area in Figure 6 implies entry (=1). In the clear case of entry (the brightest area), a potential entrant is more likely to bear the fixed cost of entry when its initial capital, k_0, is relatively larger than its debt, b_0. It follows that the slope of the entry threshold, where the color map changes into the brightest color, represents the maximum leverage that the actual entrants can take. The area of entry becomes larger when the initial productivity of potential entrants is higher, so the dark colored areas between 0 and 1 illustrate the entry decisions across potential entrants with different ϵ_0. That is, more firms enter the economy in the areas with lower k_0 and higher b_0 as initial productivity increases. Since the share of potential entrants with high ϵ_0 is relatively small, the average entry probability gets lower at higher productivity levels. Moreover, the slope of the entry threshold in the figure becomes steeper as the initial productivity increases. This implies that the entrants with high ϵ_0 are allowed to take more leverage when they decide to enter. Such firms have strong growth potential over time, while the persistence and the relative dispersion of the underlying productivity distribution jointly determine the severity of financial frictions at the micro level.

4 Targeted Credit Policies

We next examine the aggregate consequences of credit subsidies. First, we provide a brief overview of our policy experiments in the model. In Section 4.2, we discuss the aggregate changes following credit subsidy policies in general equilibrium. Section 4.3 provides a step-by-step analysis of our results by isolating the moving forces in the model. We look at changes in firm dynamics in Section 4.4.

4.1 Overview of Counterfactual Exercises

Our counterfactual policy exercise is to compare the pre-intervention equilibrium described in the previous section, *(benchmark)*, and the post-intervention equilibrium, which is the new equilibrium reached by the economy after implementation of credit subsidy policies. That is, we measure the equilibrium changes in the aggregate economy under each policy relative to the benchmark.

Credit Subsidies Policy implementation works as follows. Suppose that the government selects a group of target firms and provides them with credit subsidies. If a firm faces binding collateral constraint, then the credit subsidizing policy helps it to achieve the optimal level of capital stock, by setting $\theta = q^{-1} > \theta_{ss}$, where θ_{ss} is the value at the benchmark.[26] We assume that the government can exactly identify firms with binding borrowing constraints and help only such firms. Therefore, the actual number of subsidized firms is a subset of each targeted group.[27] We consider two different target groups, small firms (*size-dependent policy*) and young firms (*age-dependent policy*). The *size-dependent policy* selects small-medium sized firms (SME), defined as those hiring fewer than 500 employees, whereas the *age-dependent policy* selects firms with age between 0 and 4.[28] The rest of the firms that are not given credit subsidies may be constrained by the borrowing limits with $\theta = \theta_{ss}$. The other model parameter values remain the same as in the benchmark.

We also attempt to relate our policy experiments to an existing policy.[29] One prominent policy that aims at helping young or small firms is the 7(a) Loan Guaranty Program by the US SBA. It is the most commonly used financing opportunity for small businesses in the US, and approved loans under the program can be flexibly spent on working capital, equipment, debt refinancing, and so forth. As shown in the table below, both the gross amount of approved loans and the average size of loans have been steadily growing over time, whereas the total number of loan approved has been somewhat stagnant.

SBA 7(a) Loan Program

	2008	2009	2010	2011	2012	2013	2014	2015	avg.
loan approved, \$bil.	12.819	9.264	12.424	19.703	15.256	18.061	19.446	23.884	16.357
no. of loans approved	69441	41273	46922	53688	44358	46389	52044	63460	52197
avg. loan, \$mil.	0.185	0.224	0.265	0.367	0.344	0.389	0.374	0.376	–

Note: The above figures are from the SBA online tables on *Loan Program Performance* (www.sba.gov/about-sba/sba-performance), as of March 2018. All observations are fiscal years.

Since there are several key differences between the SBA loan program and our coun-

[26] The government subsidizes only the additionally required investment for the targeted firms such that they can obtain an efficient scale of production at a given level of productivity. Catherine et al. (2018) also consider the similar type of counterfactuals. We include the results from modest increases in θ in the appendix.

[27] This is a different approach from that taken by Guner, Ventura, and Xu (2008). They investigate the effects of distortionary taxes on factor input uses. Once a firm falls into the targeted group in their model, its size is always restricted by such policies.

[28] Our definitions of small and young firms are consistent with those of Fort et al. (2013).

[29] We thank an anonymous referee for encouraging us to examine how an actual policy translates into the model.

terfactual policies, a direct mapping of the policy into our model economy is infeasible.[30] Instead, we attempt an indirect but useful counterpart of the actual policy in the model. The basic idea of this exercise is to count the number of firms that are affected by the SBA program, and then to find a comparable case of the size-dependent policy that adjusts θ for SMEs in our model.

According to the previous table, the average loan approval per year is 52,197 between 2008 and 2015. This is about 1.02 percent of all private firms in 2015 BDS, implying that a very small number of SMEs were able to obtain financial assistance using the SBA loan program. We assume that these firms were previously at their borrowing limit, which are identified as Type-2 firms in our model. Noting that about 9.92 percent of firms are Type-2 in the benchmark economy, we find a corresponding policy for SMEs in the model which reduces Type-2 firms by 1.02 percent. The imputed size of policy is to raise θ by 5 percent (0.82 to 0.861) for SMEs in the model. We regard this specific policy intervention as the model counterpart to the actual policy implemented in the US. The main difference of our policy from the SBA program is that we provide economy-wide credit subsidies to all Type-2 firms which leads to a fall in the number of such firms after the policy.

Measuring Policy Effects We assume that subsidized capital is financed by lump-sum cash transfers from households. This assumption allows us to isolate and highlight the unintended effects of credit subsidies through general equilibrium price adjustments. Importantly, the total amount of cash transfers will be determined in equilibrium as policies affect the distribution of firms and change the level of unconstrained capital, $K^w(\epsilon)$. To compute the total cash transfers under a certain policy, we identify the firms with binding borrowing constraints and then calculate the gap between those firms' efficient investment, i, and the constrained level of investment without the policy, i^B. The total cash transfer, rt, is computed by aggregating the gap between i and i^B:

$$rt(p) = \int_{\mu^B} (i - i^B) d\mu^B,$$

where μ^B denotes the measure of subsidized firms with binding borrowing constraints on $\mu(k, b, \epsilon)$. Notice that rt can be large because our policy intervention is economy-wide, in contrast to actual policies with low take-up rates.[31]

[30] There is substantial heterogeneity in approved loan rates and maturity terms across firms. Moreover, the SBA has its own size standard for eligibility, and the employment size limit for being a small business varies from 500 to 1500 by industry. This limits our ability to construct a more direct and realistic mapping.

[31] Buera, Kaboski, and Shin (2017) also discuss the observed low take-up rates of microfinance programs in developing economies.

We measure the aggregate gains, relative to the benchmark economy, in the measured total factor productivity, ΔTFP, following each credit subsidy policy. We calculate TFP as the Solow residual, $Y/(K^{\alpha}N^{\nu})$, given aggregate output (Y), capital (K), and labor (N) while fixing α and ν values at the benchmark. As will be discussed below, any policy intervention in our model economy involves changes in the equilibrium number of firms. Due to DRS production technology, TFP generally falls in the total number of firms given the same aggregate quantities. To control for the differences in the number of firms across policy regimes, we also measure the average productivity gains per firm, ΔTFP_{μ}, which can be calculated by re-scaling the aggregate variables by the measure of firms. In addition, note that the required cash transfer in each policy differs when the firm distribution changes in equilibrium. We normalize ΔTFP by the total amount of credit subsidy, $\Delta TFP/rt$, to measure the average TFP gain per unit of credit subsidy. This quantifies how effective each policy is in terms of resolving resource misallocation across firms.

Lastly, we compare the welfare consequences from different policies aimed at resolving financial frictions. Following the standard approach, we measure consumption equivalence variations (CEV) relative to the benchmark economy.[32]

4.2 Aggregate Results

This sub-section presents the results from our counterfactual analysis of the age- and size-dependent policies and those from the model counterpart to the SBA program described above. We examine the aggregate effects of each credit subsidy policy and discuss the direct effect and the indirect general equilibrium effects.

4.2.1 Direct and Indirect Effects of Policies

As in Table 3, both age- and size-dependent policies largely improve aggregate outcomes in the economy. Specifically, the age-dependent policy for young firms raises aggregate consumption and capital, respectively, by 5.6 and 11.5 percent from the benchmark, while output slightly falls. In the following, we explain that this fall in aggregate output is mainly from the huge decrease in the number of firms after the age-dependent policy is implemented. Aggregate output under the size-dependent policy that targets small-medium sized firms (SMEs), on the other hand, increases by 2.7 percent from its

[32] Hopenhayn and Rogerson (1993) use a similar approach to investigate the welfare effects of firing costs.

pre-policy equilibrium. In addition, our welfare measurement (CEV) increases by more than 9 percent following these credit subsidy policies This is mainly due to the rise in equilibrium consumption and the fall in employment in both cases. Because we abstract from policy-related distortions in our policy experiments, we view the aggregate improvements in Table 3 as the upper bounds of the positive impacts from the economy-wide credit subsidizing policies.

Table 3 : Aggregate Results from Policy Experiments

Policy Counterfactual: Aggregates			
	Benchmark	Age-dependent (age 0 to 4, $\theta = q^{-1}$)	Size-dependent (SMEs, $\theta = q^{-1}$)
consumption	100 (0.1876)	105.60	106.29
capital	100 (0.5744)	111.54	113.04
output	100 (0.2584)	99.88	102.71
employment	100 (0.3334)	94.54	96.55
debt	100 (0.3238)	147.56	144.66
cash-on-hand	100 (0.8020)	80.27	87.68
firms (μ)	1.0001	0.5193	0.6802
endo. exit rate	0.0140	0.0430	0.0203
entrants rel. size	0.0331	0.0197	0.0196
cash transfer (rt)	–	0.2282	0.2003
subsidized firms (μ^B)	–	0.0558	0.0555
Type-2 share	0.0992	0.0117	0.0151
$\Delta TFP(\%)$	(0.5834)	0.1885	1.3541
$\Delta TFP/rt$	–	0.0048	0.0394
$\Delta TFP_\mu(\%)$	–	8.3819	6.1536
CEV(%)	–	10.4700	9.3700

Note: In the top panel, we normalize aggregate quantities to 100 in the benchmark, and the values in the parentheses are the corresponding absolute levels. rt is the required cash transfer in each policy. ΔTFP refers to the relative change in the measured total factor productivity from the benchmark. ΔTFP_μ is the productivity gain per firm. CEV denotes the consumption equivalent measure of welfare.

The above aggregate improvements are mainly due to the reduced resource misallocation across firms. In our model, collateral constraints prevent small but productive firms from undertaking investment at their desired level. Insufficient capital in these firms reduces aggregate TFP. Therefore, providing credit subsidies to such firms resolves capital misallocation, which in turn leads to increased aggregate productivity. In Table 3, this first-order impact is shown by the fall in the share of Type-2 firms, thus raising the measured TFP (ΔTFP) following each targeted policy. The productivity improvement from the age-dependent policy is 0.19 percent relative to the benchmark, and that from the size-dependent policy is 1.35 percent. These gains are obtained by subsidizing firms with binding borrowing constraints, which accounts for about 8 to 10 percent of the entire population (μ^B/μ). We also normalize the productivity gains by the size of cash transfers from households, ($\Delta TFP/rt$). It follows that the size-dependent policy leads to higher marginal gains per transfer in productivity than the age-dependent policy does.

Notice that the aggregate cash transfers, (rt), under both policies are very large, accounting for more than 30 percent of aggregate capital. One reason is that our credit subsidies are available economy-wide for all young or small firms with binding borrowing constraints. This is not exactly the same in actual policies that typically support a small number of firms in an economy as in the SBA program, and we show the results from such cases later in this sub-section. Another reason is that our policy counterfactual is rather extreme in the size of intervention per firm. In both age- and size-dependent policies, we totally remove collateral constraints for the affected firms. Under such large policy interventions, there are substantial increases in the average productivity of incumbents as initially intended, while the gains in TFP are much smaller. This discrepancy is due the indirect effects of such policies in general equilibrium, as we discuss below.

The number of firms in production (μ) drastically falls following each policy implementation, as reported in Table 3. Since the number of firms also affects the measured TFP in each policy regime, we separately calculate the average productivity gain per firm, ΔTFP_μ. The productivity gain per firm is 8.38 percent under the age-dependent policy and 6.15 percent under the size-dependent policy targeting SMEs. It follows that targeting young firms results in relatively higher productivity gain for an average incumbent, but the associated improvement in measured TFP is rather modest. Therefore, our results from measuring different gains in productivity, whether it is the average per firm or per cash transfer, imply that each targeted policy has an edge over the other. This sheds light on the importance of targeting the group of firms whose marginal gains are large in practice.

In addition to the above direct effects from undoing capital misallocation, providing credit subsidies involves indirect effects in general equilibrium. Following the targeted policies, the equilibrium changes in relative prices adjust the entry and exit margins, which eventually determines the number of firms in the economy. We now provide discussions about these indirect channels of policies.

We begin with the extensive margins of firm entry and exit. In labor markets, due to the boosted aggregate productivity, higher labor input by the subsidized firms leads to an increase in the equilibrium wage rate. The rise in factor price puts more pressure on firms with low profitability to exit. This mechanism is represented by the higher endogenous exit rates under the policies in Table 3, ranging from 2.0 to 4.3 percent in each period.

Moreover, the higher equilibrium wage rate also affects the entry margin of firm dynamics, which either strengthens or weakens the selection among potential entrants. That is, potential firms in the model observe higher profitability under each policy that relaxes borrowing constraints, but the increased cost of production prevents those with low productivity from entering. These two opposing effects lead to an ambiguous prediction on the entry margin following each policy, and we show the corresponding overall results quantitatively. Table 3 demonstrates that both targeted policies induce more small firms to enter the economy by paying the fixed entry cost. The average employment size of entrants is about 60 percent (0.0197 and 0.0196) of that in the benchmark (0.0331).[33] Therefore, we show that a credit subsidy targeting young or small firms rather weakens the selection among potential entrants in equilibrium, even with the rise in the wage rate. In the next sub-section, we also look at the relative size of this selection effect while holding the equilibrium adjustments in exit margin and wage rate fixed.

Figure 7 illustrates the change in the entry margin after the size-dependent policy. It clearly shows that entry occurs at any initial capital and debt combination, (k_0, b_0), with positive probability, in contrast to the results shown in Figure 6 for the benchmark economy. Thus, the average size of entrants significantly falls, as shown in Table 3. Moreover, Figure 7 shows that the entry threshold changes as the credit subsidy is implemented. For instance, at $b_0 = 0$, the capital threshold for the brightest area falls from 0.0992 to 0.0949. Since the slopes of the threshold lines under the size-dependent policy also become steeper, it follows that the marginal entrants take on relatively more leverage. Because these entrants are relatively smaller, they are also likely to face tighter collateral constraints in addition to the fixed operating cost in the model. Combined with the higher leverage, this raises the exit hazard of age 0 firms disproportionately, thus contributing to the increased exit rates as shown in Table 3.

From the adjustments in the extensive margins, the number of firms falls in equilibrium, which in turn depresses aggregate productivity. Due to DRS production technology at the firm level, the number of producing firms is a non-trivial factor that affects measured aggregate total factor productivity. As already shown in Table 3, the total number of firms decreases by 32 to 48 percent relative to the benchmark economy. Fewer firms in operation following the targeted policies dampen the aggregate gains in productivity,

[33] This is qualitatively consistent with the findings in Banerjee et al. (2015). They find that the new businesses supported by a microfinance program in India tend to have significantly smaller size.

and this is why we observe only modest improvements in the measured TFP (ΔTFP). The negative impact on TFP due to the fall in the number of firms can be represented by the difference of the relative gains in TFP (ΔTFP) and in the average productivity per firm (ΔTFP_{μ}). The age-dependent policy is relatively more successful in boosting average firm productivity by more than 8 percent, but this gain almost disappears by losing more firms in equilibrium.

Notice that the decreases in the number of firms are quantitatively large following each credit subsidizing policy. One reason is that the size of policy intervention is enormous in our policy experiments, as noted earlier, in terms of both the number of targeted firms and the amount of individual subsidies. In addition, the skewed firm size distribution that we reproduce in our benchmark economy features a large number of small firms. Some of these firms are productive but financially constrained, while the others are not. Since the latter firms are not targeted, they are vulnerable to a rise in relative prices following a policy and thus more likely to choose to exit. It follows that this cleansing effect of the policy quantitatively depends on the underlying firm-level productivity in a model. As will be shown in Section 5, our joint consideration of the endogenous extensive margins and the non-Gaussian firm productivity distribution is crucial in determining the aggregate elasticity of an economy following a policy.

So far, we are able to identify both direct and indirect effects of credit subsidy policies targeting small and young firms. As expected, a targeted policy directly resolves the resource misallocation faced by small or young firms. However, such policies also have unintended effects that indirectly emerge from the equilibrium price channel. In particular, the increased price under each policy has three effects at the firm level that in turn affect the aggregate results. First, it reduces the efficient scales of production for all firms. Next, the rise in price also adjusts the firm entry and exit margins to determine the equilibrium number of firms following the introduction of a policy. The last effect comes from the reduction in the total number of firms, which lowers the aggregate productivity gains. Together, these direct and indirect effects jointly quantify the aggregate outcome of a credit subsidy policy. Our results suggest that the unintended effects are quantitatively substantial when such policies are present.

4.2.2 The SBA Loan Program

We report the aggregate results of our model counterpart to the SBA program. Table 4 shows the predicted effects of the SBA-counterpart policy, ($\theta \uparrow 5\%$), and our previous results from the size-dependent policy that totally eliminates borrowing limits for SMEs,

($\theta = q^{-1}$).

Table 4 : Aggregate Results, Size-dependent Policy and SBA

Policy Counterfactual: Aggregates, SBA Policy			
	Benchmark	SBA ($\theta \uparrow 5\%$)	Size-dependent ($\theta = q^{-1}$)
consumption	100 (0.1876)	100.59	106.29
capital	100 (0.5744)	100.70	113.04
output	100 (0.2584)	99.96	102.71
employment	100 (0.3334)	99.37	96.55
debt	100 (0.3238)	104.01	144.66
cash-on-hand	100 (0.8020)	97.85	87.68
firms (μ)	1.0001	0.9589	0.6802
endo. exit rate	0.0140	0.0154	0.0203
entrants rel. size	0.0331	0.0316	0.0196
Type-2 share	0.0992	0.0890	0.0151
$\Delta TFP(\%)$	(0.5834)	0.1371	1.3541
$\Delta TFP_{\mu}(\%)$	–	0.6514	6.1536
CEV(%)	–	1.1100	9.3700

Note: In the top panel, we normalize aggregate quantities to 100 in the benchmark, and the values in the parentheses are the corresponding absolute levels. ΔTFP refers to the relative change in the measured total factor productivity from the benchmark. ΔTFP_{μ} is the productivity gain per firm. CEV denotes the consumption equivalent measure of welfare. The first and third columns are reproduced from Table 3, for comparison.

Overall, the case of raising borrowing limits by 5 percent leads to a similar aggregate outcome but of smaller magnitude, when compared to the case of completely removing them. First, there exists a small aggregate improvement in the economy due to the increase in TFP by 0.14 percent, following such limited credit subsidies. We still observe substantial indirect effects of the policy, as represented by the difference between ΔTFP and ΔTFP_{μ}. Again, this is due to the fall in the number of firms which is more than 4 percent less than the benchmark economy. In contrast, the rise in wage rate is only about 0.58 percent from the benchmark. Our results suggest that firms sensitively respond to credit policies through adjustments in the extensive margins, while changes in equilibrium prices are relatively small. Due to the fall in the number of firms, aggregate output and employment decrease despite a rise in the productivity of incumbents.

As previously discussed in detail, our analysis demonstrates the importance of considering the unintended effects of a policy that aims to promote the reallocation of resources among firms. Although the above mapping to the actual policy is stylized, we provide one possible method of evaluating the long-run aggregate impact of existing small business policies.

4.3 Inspecting the Mechanism

The indirect effects of the credit subsidy policies are quantitatively important, offsetting potential gains in aggregate productivity that would otherwise arise. There are different channels operating when a credit subsidizing policy is implemented. These channels include reallocation among incumbent firms as well as adjustments in the extensive margins, following an equilibrium change in prices. In this sub-subsection, we decompose the total effect of a policy to isolate the net effect from each channel.[34] To simply our analysis, we only consider the SBA-counterpart policy that provides credit subsidies to SMEs that are financially constrained.

First, the following table summarizes our procedure of decomposing the total effect. Starting from the initial equilibrium before the policy is implemented, *(Benchmark)*, we introduce one channel at a time. In particular, we consider 3 different stages through which the economy reaches the new equilibrium, *(GE)*, after the policy. For convenience, we name these stages as *(Model A)*, *(Model B-1)*, and *(Model B-2)*. Our choice of these stages is motivated by our previous analysis of the indirect effects of the policy in general equilibrium.

Isolating the Effects, Multiple Stages, SBA Policy

	Model Changes from (Benchmark)				
	(Benchmark)	(Model A)	(Model B-1)	(Model B-2)	(GE)
wage	–	–	–	–	✓
credit subsidies	–	✓	✓	✓	✓
entry margin	–	–	✓	✓	✓
exit margin	–	–	–	–	✓
firm measure	1.0001	1.0001	1.0001	0.9589	0.9589

Note: (Benchmark) is the initial equilibrium without a policy, and (GE) is the new equilibrium after the policy. Once the policy is implemented, while fixing the wage rate, we distinguish the intermediate stages by whether the entry margin is adjusted or firm measure is re-scaled.

As shown in the first row of the table, each of the above steps maintains the initial equilibrium prices in (Benchmark) which are finally replaced in (GE). In (Model A), we allow reallocation of capital among incumbents following the introduction of subsidy, while holding the distribution of firms fixed. Since there is no change in the entry and

[34] We are grateful to an anonymous referee for suggesting this exercise by which the quality of our quantitative analysis could be substantially improved.

exit margins in this step, the measure of firms also remains the same as in (Benchmark). In this way, we can measure the net productivity gains only from subsidizing financially constrained firms. Next, in (Model B-1), we allow adjustments in the entry margin that follows the policy. Since we keep relative prices fixed at (Benchmark), the credit subsidy induces more entry by small and financially constrained entrants. To capture the net selection effect upon entry, we calculate the measured TFP for a unit measure of firms in (Model B-1). In contrast, (Model B-2) is the case when we re-scale the firm distribution of (Model B-1) to match the number of firms at (GE) in which the changes in relative prices are introduced. Notice that the number of firms drops when we switch from (Model B-1) to (Model B-2), while the composition of firm types remains the same. Thus, the associated change in TFP across these two stages implies the net effect from the change in the number of production units. In the meantime, the TFP change from (Model B-2) to (GE) measures the effect of higher wage on production scale, after the policy. Lastly, the shift from (Model B-1) to (GE) represents the overall cleansing effect in general equilibrium, combining the previous two effects with the adjustment in the exit margin.

In Table 5, we report the relative changes in TFP after introducing the SBA policy to the benchmark economy. First, the net reallocation effect among the incumbents is positive, raising the measured TFP by 1.05 percent. This is the direct effect of the credit subsidy that partly resolves the existing resource misallocation across firms. TFP slightly falls by about 0.4 percent, when we consider the change from (Model A) to (Model B-1). This is because the subsidy induces relatively less-productive entry, while the exit margin remains the same as in (Benchmark). As explained above, the shift from (Model B-1) to (Model B-2) measures the net effect from the fall in the number of firms. This effect is large enough to nearly offset the previous positive TFP gains from the policy. It turns out that this fall in the number of firms is dominant, so that we only observe a modest increase in overall aggregate productivity. Once firms are allowed to operate at the new equilibrium price along with the exit margin adjustment, (Model B-2) to (GE), there is no change in TFP.

We further observe that the overall cleansing effect from (Model B-1) to (GE) is substantially negative (-0.51 percent), which almost offsets positive TFP gains from the policy. This is because the financial friction in our model also plays a role in eliminating less productive firms through the endogenous exit margin. Once the credit subsidy is implemented, however, this cleansing mechanism of the collateral constraint is weakened so that relatively less productive firms survive longer in the economy. The overall magnitude of the cleansing effect is endogenously determined by adjustments in both entry and

Table 5 : TFP Changes across Intermediate Stages, SBA policy

Isolating the Effects, SBA policy					
	(Benchmark)	(Model A)	(Model B-1)	(Model B-2)	(GE)
(Pareto ϵ)					
firm measure	1.0001	1.0001	1.0001	0.9589	0.9589
TFP	0.5834	0.5895	0.5872	0.5842	0.5842
$\Delta TFP(\%)$	–	1.0456	0.6514	0.1371	0.1371

Note: We calculate TFP in each stage, while fixing α and nu values at (Benchmark). $\Delta TFP(\%)$ denotes the percentage deviation of TFP from (Benchmark).

exit margins, which eventually affects the equilibrium number of firms in the economy, as discussed in the previous sub-section. Our quantitative results suggest that individual effects of these channels are large while offsetting each other. It follows that there can be a small increase in aggregate productivity from a credit subsidizing policy, when we take all the above indirect effects into account in a general equilibrium environment.

4.4 Firm Dynamics

In Section 4.2, we consider credit subsidizing policies oriented towards different targets, either young or small firms, and hence having disparate impacts. It is evident that such policies affect firm-level decisions at different stages of the lifecycle. In this sub-section, we examine how targeted credit subsidies reshape firm dynamics by looking at the evolution of average size and productivity in a cohort over time.

Figure 8 compares the average firm dynamics under different policies, where the values in the top panels are in relative to their respective levels at age 20. The upper-left panel of the figure shows the typical growth pattern of entrants in our benchmark economy (*blue line with dots*). First, firms, on average, start small. The relative size of entrants is about one-fourth of that of mature firms of age 20. Upon entry, entrants start accumulating capital and gradually become larger as they age. As they become older, selection forces unproductive incumbents in cohort to exit the economy. Hence, as shown in the upper-right panel of Figure 8, the average productivity of firms increases over time, whereas the productivity of age-0 firms in our benchmark economy is about 20 percent lower than that of age-20 firms.

Under the age-dependent policy, all firms between age 0 and 4 can achieve optimal capital investment as collateral constraints are entirely lifted. This is reflected in the upper-left panel of Figure 8, where the policy (*red-dashed line with dots*) allows firms aged 1 to 5 to adopt their unconstrained levels of capital. To reach unconstrained capital at age 1, entrants take on high leverage at age 0. Thus, these young firms have their largest borrowing levels at entry and then gradually de-leverage over time, conditional on survival (lower-left panel). Due to the large inflow of small entrants under this policy, as discussed earlier, the productivity of age 0 firms is slightly lower than that of the benchmark (upper-right panel). In addition, firms older than age 4 become ineligible for the age-dependent policy, so their average size reverts back to the levels when borrowing constraints impact capital choices. However, since those firms were able to accumulate more cash-on-hand under this policy, they tend to save faster in terms of financial assets after age 5, as illustrated by the steeper fall in debt over time. This implies that, at age 5, firms are better prepared for idiosyncratic risks due to their lower leverage ratio, which is a relatively sound financial position. Thus, we confirm that our age-dependent policy supports young-productive firms by allowing them to survive longer, similar to selectively protecting infant firms.[35]

In contrast, the case of subsidizing SMEs (*solid green line*), also shown in Figure 8, does not deliver the same patterns of firm growth observed under the age-dependent policy. This is because the size-dependent policy always allows small-productive firms to effectively finance their desired investment whenever they become Type-2, regardless of age. Since most entrants in our model are small, the policy still accelerates the growth of average firm size and productivity in comparison to the benchmark. This enables young firms to immediately increase their leverage upon entry, but the pattern of de-leveraging after age 5 is similar to that in the benchmark economy. Moreover, the indirect effect on the entry margin still exists. As in the case of age-dependent policy, the credit subsidy for SMEs induces more entry by smaller firms in equilibrium. This slightly lowers the productivity of firms at age 0 (top-right panel) which then immediately rises to the levels of age 20 firms, due to the policy. Recall that the top panels in Figure 8 are relative to the values at age 20 in each economy. In fact, the average productivity of a cohort under the size-dependent policy always reaches a higher absolute value than that in the benchmark, starting from age 1. It also follows that subsidizing only SMEs is a special case of removing financial frictions in the entire economy, because the collateral constraint we consider mainly limits borrowing by firms with insufficient cash-on-hand in the model.[36]

[35] This view is shared by Moll (2014), who distinguishes between the loss from misallocation at the steady state and that during a transition. With persistent productivity shocks, Moll shows that the loss at the steady state is relatively small, while transition dynamics are more gradual. In this regard, our policy exercises consider the possibility of expediting the transition process by allowing firms to self-finance.

[36] In the appendix, we show the results from a non-targeted policy which is the case without financial frictions in the model.

5 The Importance of Capturing the Firm Size Distribution

In Section 3, we aligned our model to be consistent with the empirical firm size distribution, in the presence of endogenous margins of firm entry and exit. In this section, we show how quantitative predictions of a credit subsidy policy can be changed when we abstract from those two model elements.

5.1 Firm Size Distribution and Extensive Margins

From our policy experiments in a general equilibrium environment, we have shown that directly resolving the resource misallocation faced by small or young firms enhances productivity. However, this direct effect of a policy may be offset by the accompanying indirect effects in equilibrium, which leads to modest TFP gains in the end.

Later in this section, we show that the aggregate productivity gain from credit subsidy policies could even be negative when a model ignores the observed heterogeneity in firm size in the data. That is, the evaluation of such policies may be significantly different and misleading, when firm-level productivity shocks are assumed to be Gaussian, a standard modeling assumption, in contrast to our benchmark model with Pareto-distributed productivity. As we clarify below, the model with a standard AR(1) process cannot capture the right tail of firm size distribution. It follows that such model ignores firms that are currently small due collateral constraints but have strong growth potential once collateral constraints are lifted, for which there are large potential gains from policies aimed at relaxing borrowing limits. Without such firms, the direct effect of credit subsidies becomes relatively weaker than the negative indirect effects that arise from equilibrium adjustments in the extensive margins. If the latter is dominant, a policy may result in a negative TFP gain in models without a realistic firm size distribution. It also follows that such indirect effects are predicted to be much smaller in models only with exogenous margins of firm entry and exit.

To examine the quantitative importance of our joint consideration of skewed firm-level productivity and endogenous extensive margins, we depart from our benchmark economy along two dimensions: log-normally distributed productivity and exogenous entry and exit. In each different model specification, we re-calibrate the model parameter values at the respective steady state to be consistent with the observed moments in Table 2.[37]

[37] With log-normally distributed productivity, the model firm size distribution is not consistent with its empirical counterpart. So, our re-calibration for such models is not intended to completely reproduce

For the alternative specification of firm-level productivity, we consider a log AR(1) process for ϵ, which replaces the Pareto distribution. Specifically, we assume $\log \epsilon' = \rho_\epsilon \log \epsilon + \eta'_\epsilon$ with $\eta'_\epsilon \sim \mathcal{N}(0, \sigma^2_\eta)$, which is a standard modeling choice adopted by recent studies of heterogeneous firm models. To make the log-normal ϵ process comparable to our benchmark, we simulate the Pareto ϵ process and fit the assumed AR (1) process to set the values of ρ_ϵ and σ_η.[38] Using these two parameters, we discretize the log-normal AR(1) productivity process on a grid of 13 points, as we did in the benchmark parameterization. Although both Pareto and log-normal productivity distributions lead to the same persistence and volatility at the individual firm level, the population shares of firms across productivity grid points are largely different, as shown in the left panel of Figure 10.

For a model with exogenous entry and exit, we simply set the value of π_d, the exogenous exit rate in each period, to be the same as the total exit rate in the benchmark economy (11.4 percent). In addition, we allow all potential entrants with mass $M^e = \pi_d$ to start production without paying the fixed entry cost, while matching the observed employment size of entrants in the data. This model environment abstracts from the endogenous cleansing effect of replacing unproductive incumbents with new firms, which we discussed in the previous section. Hence, the model with only exogenous extensive margins leads to a relatively higher population share of Type-2 firms whose collateral constraints are binding at the pre-intervention equilibrium. In fact, such a model has about 29 percent of firms as Type-2, which is almost triple the amount in our benchmark. Thus, any policy targeted toward such firms will deliver quantitatively different aggregate results while this alternative model still captures the moments reported in Table 2. In this regard, we highlight the importance of endogenous changes in the extensive margins when credit subsidy policies are introduced.

5.2 Comparison of Aggregate Results

We report the aggregate results from our policy counterfactuals in the models with different specifications for firm-level productivity and extensive margins. In the following, we focus only on the case of the size-dependent credit subsidy policy. Table 6 presents the results at pre- and post-intervention equilibria; to facilitate comparisons, the earlier

the size distribution moments in the bottom panel of Table 2.

[38] Figure 9 illustrates the simulation of the two different productivity processes with the same mean, persistence, and volatility. In our policy counterfactual, we normalize the mean of the log AR(1) process to be 1, which is innocuous.

values in Table 3 are reproduced in the first 2 columns for our benchmark economy with Pareto ϵ and endogenous margins.

Table 6 : Aggregate Results by Model Specification

Policy Counterfactual: Aggregates, Size-dependent Policy ($\theta = q^{-1}$)

ϵ process	Bounded Pareto				Log-normal AR(1)			
entry/exit	Endogenous		Exogenous		Endogenous		Exogenous	
	SS	Size	SS	Size	SS	Size	SS	Size
consumption	(0.1876)	106.29	(0.2281)	110.65	(0.4681)	100.73	(0.5084)	104.90
capital	(0.5744)	113.04	(0.5989)	124.21	(1.5912)	102.59	(1.4665)	109.24
output	(0.2584)	102.71	(0.2693)	112.81	(0.6592)	100.09	(0.6096)	105.61
employment	(0.3334)	96.55	(0.3332)	101.95	(0.3332)	99.37	(0.3332)	100.69
firms (μ)	1.0001	0.6802	1.0000	1.0000	1.0004	0.9315	1.0000	1.0000
endo. exit rate	0.0140	0.0203	–	–	0.0141	0.0321	–	–
entrants rel. size	0.0331	0.0196	0.0331	0.0178	0.0331	0.0325	0.0331	0.0247
$\Delta TFP(\%)$	(0.5834)	1.3541	(0.6005)	4.9792	(1.1192)	-0.2412	(1.0589)	2.6065
CEV(%)	–	9.3700	–	9.1300	–	1.2600	–	4.3800

Note: *SS* refers the initial steady state equilibrium, and *Size* is the new equilibrium reached after the size-dependent policy. In the top panel, we first normalize aggregate quantities to 100 in each SS and report their relative changes under the policy. ΔTFP refers to the relative change in the measured total factor productivity from each steady state. CEV denotes the consumption equivalent measure of welfare. We use re-calibrated values of α and ψ to compute TFP and CEV in each model specification.

First, in the first 4 columns of the table, we compare the aggregate consequences of the size-dependent policy *with and without the endogenous margins* of firm entry and exit. In contrast to our benchmark case, the policy counterfactual for the model with exogenous entry and exit exhibits larger positive changes for all the aggregate variables (columns 3 and 4), apart from the number of firms (μ) which remains constant. Without a decline in the total number of firms in operation in the economy, the aggregate productivity gain is larger in this model, about 4.98 percent, in contrast to the benchmark case with a 1.35 percent gain. This result is intuitive because there is no equilibrium adjustment in the extensive margins which negatively affects the aggregate productivity eventually. Our result highlights the importance of endogenous extensive margins in quantitatively determining the macroeconomic impact of credit subsidizing policies. It is also evident that the losses from resource misallocation due to financial frictions can vary across models with different specifications of firm entry and exit, as pointed out in recent studies.[39]

In Table 6, the same policy experiment in the model instead with a log AR(1) ϵ process

[39] Midrigan and Xu (2014) consider a two-sector model economy with financial frictions existing only in the modern sector. They consider endogenous extensive margins in their extended version of the model and find relatively small losses from resource misallocation, which is consistent with our results.

(columns 5 and 6) reveals a more striking result; the sign of the aggregate productivity gain changes from positive to negative. Specifically, the credit subsidy in this model lowers the measured TFP by 0.24 percent relative to its pre-intervention steady state.[40] This is in stark contrast to the results from our benchmark economy where both size- and age-dependent policies, at least, lead to positive gains in aggregate productivity. Moreover, it is noticeable that the negative impact on TFP emerges without any direct policy distortions because we maintain our assumption of lump-sum cash transfers across all model variations in Table 6. As a result, the aggregate improvement is quite modest in the model with log-normally distributed firm productivity, and the corresponding household welfare increases only by 1.26 percent. Recall that we adopt the Pareto-distributed ϵ process mainly to reproduce the observed dispersion and skewness in firm size distribution in the data.[41] Thus, the above comparison simply illustrates that a model without a realistic firm size distribution may give rise to inaccurate, or potentially wrong, predictions of targeted industrial policies in the long run.

The above result is largely robust across different sizes of policy intervention. To see this, we now vary the rise in θ by 5 percent and 10 percent. Table 7 reports the aggregate results from the model with log AR(1) firm productivity and endogenous entry and exit margins, following these limited subsidies. In the table, we still observe the same patterns of aggregate changes though at smaller magnitudes (columns 2 and 3). Moreover, we observe the gains in aggregate productivity dramatically falls as the size of policy intervention increases. That is, the change in TFP is almost zero in the case of raising θ by 5 percent for SMEs, and it becomes -0.11 percent when θ is 10 percent above its steady state value.[42] Since these policies still improve the average productivity of incumbents substantially, we can infer that the indirect effects in general equilibrium quantify the overall gains in aggregate productivity. Together, our results in Tables 6 and 7 demonstrate the importance of considering the skewed firm size distribution when evaluating credit subsidy policies.

To examine this disparity from the perspective of firm dynamics, we further look at how firm dynamics compare in the above models that differ in their firm productivity

[40] Similarly, the age-dependent policy in the same model results in a negative productivity gain of 0.31 percent. We provide the counterpart results of Table 6 under the age-dependent policy in the appendix.

[41] The model economy with a log AR(1) process with endogenous margins largely fails to generate a realistic firm size distribution at its steady state. The resulting population share of SMEs is 85.7 percent, in contrast to 98.5 percent in our benchmark economy, whereas their total employment share still matches the observed value in the BDS.

[42] We also observe negative TFP gains following the age-dependent policies in different sizes. See Table 14 in the appendix.

Table 7 : Aggregate Results, Model with Log-normal AR(1)

Policy Counterfactual: Aggregates, Model with log-N ϵ				
		Size-dependent Policy		
	SS	$(\theta \uparrow 5\%)$	$(\theta \uparrow 10\%)$	$(\theta = q^{-1})$
consumption	100 (0.4681)	100.30	100.53	100.73
capital	100 (1.5912)	100.68	101.50	102.59
output	100 (0.6592)	100.02	99.95	100.09
employment	100 (0.3332)	99.70	99.43	99.37
firms (μ)	1.0004	0.9752	0.9470	0.9315
endo. exit rate	0.0141	0.0172	0.0207	0.0321
entrants rel. size	0.0331	0.0326	0.0321	0.0325
$\Delta TFP(\%)$	(1.1192)	0.0089	-0.1162	-0.2412
$\Delta TFP_\mu(\%)$	–	0.3396	0.5630	0.6345
$CEV(\%)$	–	0.5500	1.0200	1.2600

Note: SS refers the initial steady state equilibrium. In the top panel, we first normalize aggregate quantities to 100 in SS and report their relative changes under the policy. ΔTFP refers to the relative change in the measured total factor productivity from SS. ΔTFP_μ is the productivity gain per firm. CEV denotes the consumption equivalent measure of welfare.

distribution. We calculate the average levels of capital and TFP for each age group of firms following the size-dependent policy considered in Table 6. Figure 11 plots the gap between pre- and post-intervention capital and TFP across age cohorts, by model specification. Overall, the dynamics of firm size and productivity display qualitatively similar patterns between the two different models. The top panel of the figure shows that credit subsidies disproportionately raise the level of capital held by young firms in relative to that by mature firms. However, the magnitude of such changes is much more pronounced in our benchmark case with Pareto-distributed firm productivity.

5.3 Factors Affecting Reallocation and Equilibrium Effects

Why are the overall policy impacts relatively small in the model with log-normally distributed productivity? As we see in the left panel of Figure 10, the right tail of the ergodic distribution of a log-normal ϵ process does not stretch out enough (the highest ϵ value is below 0.7) in comparison to the Pareto case, which has a maximum ϵ value greater than 1.1. Given the convexity of the optimal capital decision, as shown in the right panel of Figure 10, the right tail of the firm-level productivity distribution matters because the potential gain from a credit subsidy is large when firms with such high productivity are financially constrained. This is indeed the case in our comparison in Table 6, and it leads to the substantial differences in firm dynamics as shown in Figure 11. It follows, for a positive aggregate productivity gain, that firms with huge growth potential but without sufficient funds should be included in the group targeted by credit subsidy policies. As discussed earlier, this is exactly the direct effect of such policies that reallocate resources toward financially constrained firms (*RE effect*). Otherwise, as in the alternative model with log-normally distributed ϵ, the opposing indirect effect in equilibrium (*GE effect*) becomes more pervasive.

While the skewness of the productivity distribution matters for the size of the RE effect of a policy, the composition of firms in the economy affects the size of the accompanying GE effect and thus eventually determines the overall policy impacts in equilibrium. In our model, the composition of firms is characterized by firm lifecyle dynamics in the long-run. This in turn implies that the exogenous exit rate in the model plays a role in driving the GE effect as follows.[43] With a fixed probability of exogenous exit (π_d), all firms have equal chances to exit, regardless of the financial status and the level of productivity. With endogenous exit, on the other hand, financially constrained and unproductive firms are more likely to exit. Therefore, in an economy with a higher π_d, the GE effect through extensive margins is relatively weaker; and, moreover, the RE effect that hinges on the skewness of the firm size distribution becomes more important.

To see the above discussion more clearly, Table 8 decomposes the overall effect of the size-dependent policy into the RE and GE effects and examines the relative strength of these effects and their relationships with the exogenous exit rate. First, the reallocation effect is larger than the GE effect for a model with Pareto ϵ as explained above. It reflects the presence of the large number of firms with huge growth potential but being financially constrained. For all cases with different values of π_d and sizes of policy intervention, the upper panel of Table 8 shows that the relative size of GE effect against RE effect (*Ratio*) ranges from 57% to 87%. On the other hand, the bottom panel of the table indicates that the GE effect is relatively dominant for most cases with log-normal ϵ, with Ratio ranging from 97% to 124%. This is again because the model with log-normal ϵ fails to capture highly productive firms that can most benefit from a credit subsidy policy. Second, the

[43] We thank an anonymous referee for suggesting that we explore this perspective of industrial policy implications.

relative importance of the RE effect increases with π_d. As it can be seen in Table 8, Ratio in our benchmark case (column 3) is significantly larger than that in an alternative model with higher exogenous exit rate (column 6). When π_d rises, the GE effect that relies on the endogenous adjustments in the extensive margins becomes relatively smaller, as we discussed above. This result cautiously indicates that credit subsides can be more effective for industries with higher average exit rates and more homogeneous patterns of firm exit.

Table 8 : Comparison of Reallocation and GE Effects, ϵ and π_d, Size-dependent Policy

	$\pi_d = 0.10$			$\pi_d = 0.14$		
	RE, (a)	GE, (b)	Ratio, $\frac{\lvert(b)\rvert}{(a)}$	RE, (a)	GE, (b)	Ratio, $\frac{\lvert(b)\rvert}{(a)}$
(Pareto ϵ)						
$\theta \uparrow 5\%$	1.0456	-0.9085	0.8689	1.4988	-1.2548	0.8372
$\theta \uparrow 10\%$	1.3884	-1.0284	0.7407	1.8473	-1.3593	0.7358
$\theta = q^{-1}$	4.3881	-3.0340	0.6914	5.0540	-2.8930	0.5724
(log-N ϵ)						
$\theta \uparrow 5\%$	0.6433	-0.6344	0.9862	1.0136	-0.9865	0.9733
$\theta \uparrow 10\%$	0.8399	-0.9561	1.1383	1.2398	-1.2760	1.0292
$\theta = q^{-1}$	1.0007	-1.2419	1.2410	1.4118	-1.4842	1.0513

Note: *RE* is the change in TFP between (Benchmark) and (Model A), measuring the reallocation effect among incumbents, in Table 5. *GE* measures the TFP change between (Model A) and (GE), and *Ratio* is the relative size of GE effect against RE effect.

Finally, we conduct the step-by-step analysis for the model with log-normal productivity process, as in Section 4.3. As discussed before, this exercise isolates the net effects from different channels at work when a policy is implemented. Table 9 report the results, in which the top panel is taken from Table 5 for comparison. We compare the results of the SBA policy across different productivity processes.[44] First, the net productivity gain from the reallocation among incumbent firms is only 0.76 percent in the alternative model with the log-normal process (Benchmark vs. Modal A). Apparently, the direct impact of the policy becomes smaller as we miss highly productive but financially constrained firms. Moreover, the assumed productivity process affects the selection at the entry margin. By comparing (Model A) with (Model B-1), the alternative model predicts a relatively larger negative change in aggregate TFP (-0.35 percent). On the other hand, the cleansing effect remains substantial even with the log-normal productivity process so that the sign of productivity gain becomes negative (Model B-1 vs. GE). The model also predicts similar patterns of the above changes when the size of policy intervention is larger.

[44] The case of the age-dependent policy can be found in the appendix.

Table 9 : TFP Changes across Intermediate Stages, SBA policy

Isolating the Effects, SBA policy					
	(Benchmark)	(Model A)	(Model B-1)	(Model B-2)	(GE)
(Pareto ϵ, SBA Policy)					
firm measure	1.0001	1.0001	1.0001	0.9589	0.9589
TFP	0.5834	0.5895	0.5872	0.5842	0.5842
$\Delta TFP(\%)$	–	1.0456	0.6514	0.1371	0.1371
(log-N ϵ, SBA Policy)					
firm measure	1.0004	1.0004	1.0004	0.9600	0.9600
TFP	1.1192	1.1278	1.1239	1.1183	1.1188
$\Delta TFP(\%)$	–	0.7684	0.4199	-0.0804	-0.0357
(log-N ϵ, $\theta = q^{-1}$)					
firm measure	–	1.0004	1.0004	0.9315	0.9315
TFP	–	1.1304	1.1254	1.1159	1.1165
$\Delta TFP(\%)$	–	1.0007	0.5540	-0.2949	-0.2412

Note: We calculate TFP in each stage, while fixing α and nu values at (Benchmark). $\Delta TFP(\%)$ denotes the percentage deviation of TFP from (Benchmark).

In sum, we illustrate a case for which the aggregate consequences of an industrial policy may be altered when a calibrated model only targets macroeconomic moments without considering the endogenous extensive margins and the empirical firm size distribution. In other words, introducing such model elements is not only a prerequisite for being consistent with micro-level data before conducting policy counterfactual studies such as ours, but it is also crucial in examining the equilibrium aggregate results following a policy intervention. Therefore, our results indicate that the micro-level consistency of a macroeconomic model is vital to quantifying the aggregate outcomes of targeted credit policies. Moreover, this also implies that understanding the firm-level heterogeneity of an economy is essential in designing and implementing such targeted policies in practice, which we believe is missing in micro-evaluations of policies.

6 Concluding Remarks

In this paper, we presented a general equilibrium model of heterogeneous firms with collateral constraints and endogenous entry and exit. By employing a bounded Pareto distribution for firm-level productivity, our model replicates cross-sectional features of firm dynamics and firm size distribution, as seen in the BDS. We use this model to quantify the macroeconomic implications of targeted credit subsidy policies. We show that aggregate outcomes of such policies critically depend on the underlying distribution of firms and their financial status. Subsidized credit alleviates credit constraints faced by small and young firms, which helps them achieve efficient and larger scales of production. This direct effect is, however, either reinforced or offset by indirect general equilibrium effects. In particular, the indirect general equilibrium effects from declines in the number of firms in the economy dominate the direct productivity gains in a model with the standard AR(1) process, while we found a modest productivity increase by subsidy policies in our benchmark model with a bounded Pareto distribution for firm-level productivity. We view this as a cautionary tale about unintended consequences of economy-wide targeted policies that perform poorly in nurturing economic growth.

Because small and young firms account for a substantial share of employment in an economy, policies that target these firms are popular among policymakers. However, research on the aggregate implications of such industrial policies is scarce. We believe that our results demonstrate a potential pitfall in the analysis of industrial policies: failure to consider general equilibrium price movements and micro-level consistency may create misleading public debates on such policies.

Raising allocative efficiency of resources and thus enhancing aggregate productivity appear to be immediately relevant for many countries. In particular, a variety of targeted industrial policies have been implemented European and Asian economies. Quantitative investigations of such policies using our framework and micro-level firm financial data may be useful. This paper, however, did not consider any policy distortion or incentive problems which are also potentially important. This research is left for future study.

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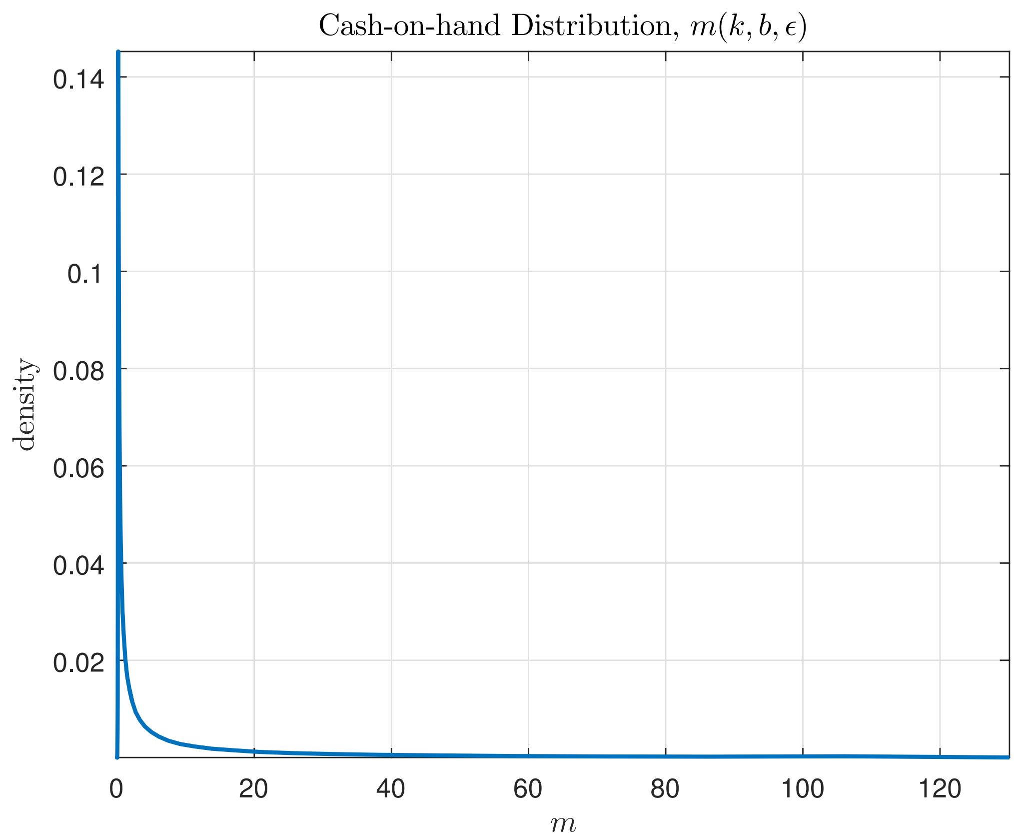

Figure 1 : The stationary distribution of cash-on-hand, $m(k, b, \epsilon)$, at the steady state (*benchmark*) of the economy.

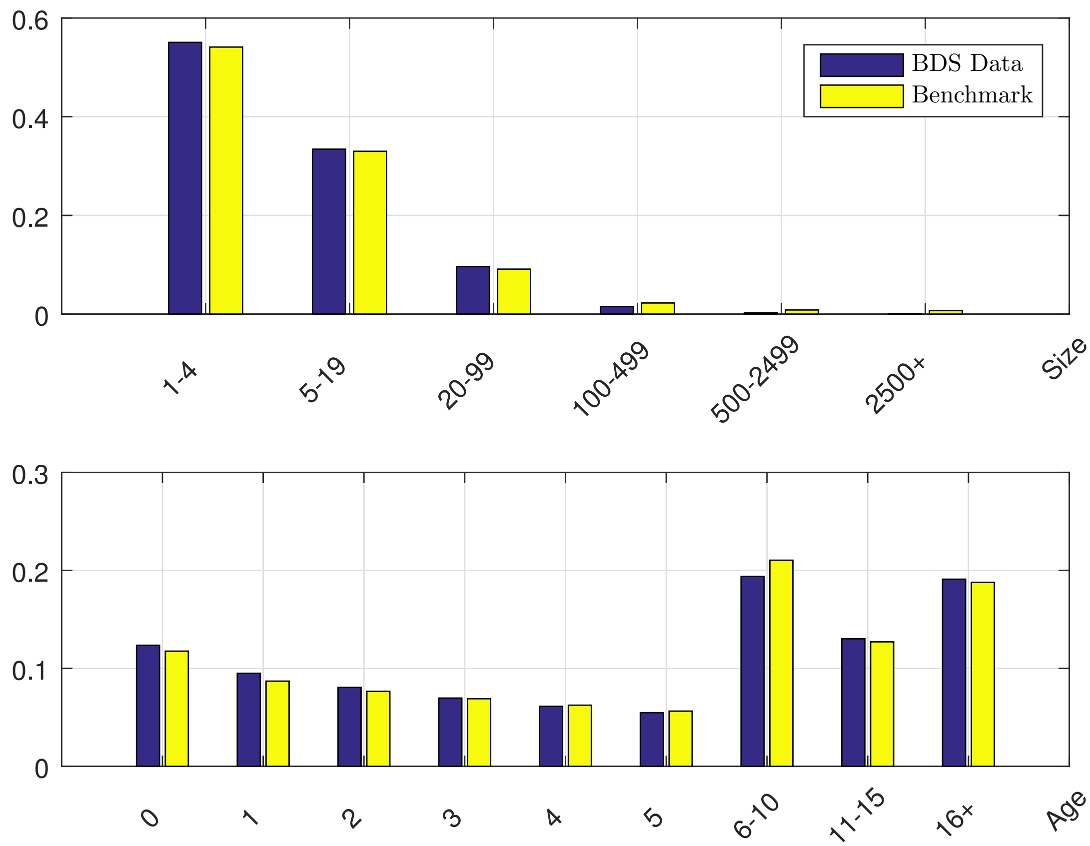

Figure 2 : Distribution of firms over size and age bins.

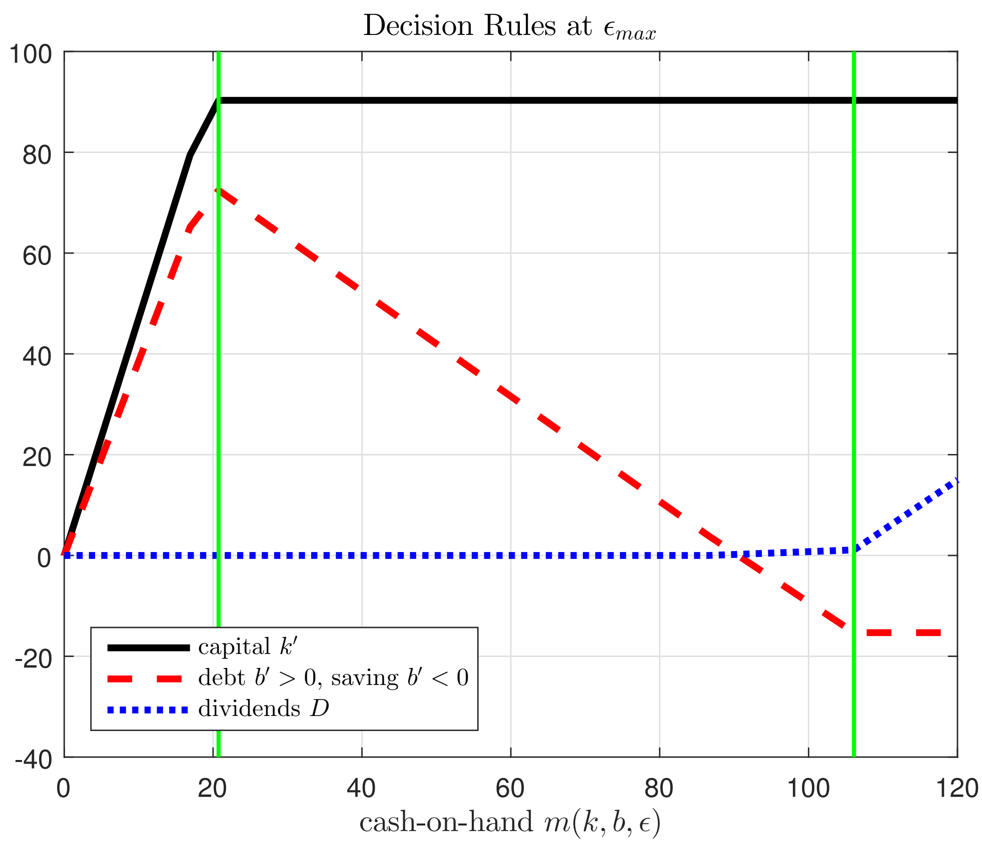

Figure 3 : Choices of capital, debt or financial savings and dividends are plotted for each level of cash-on-hand.

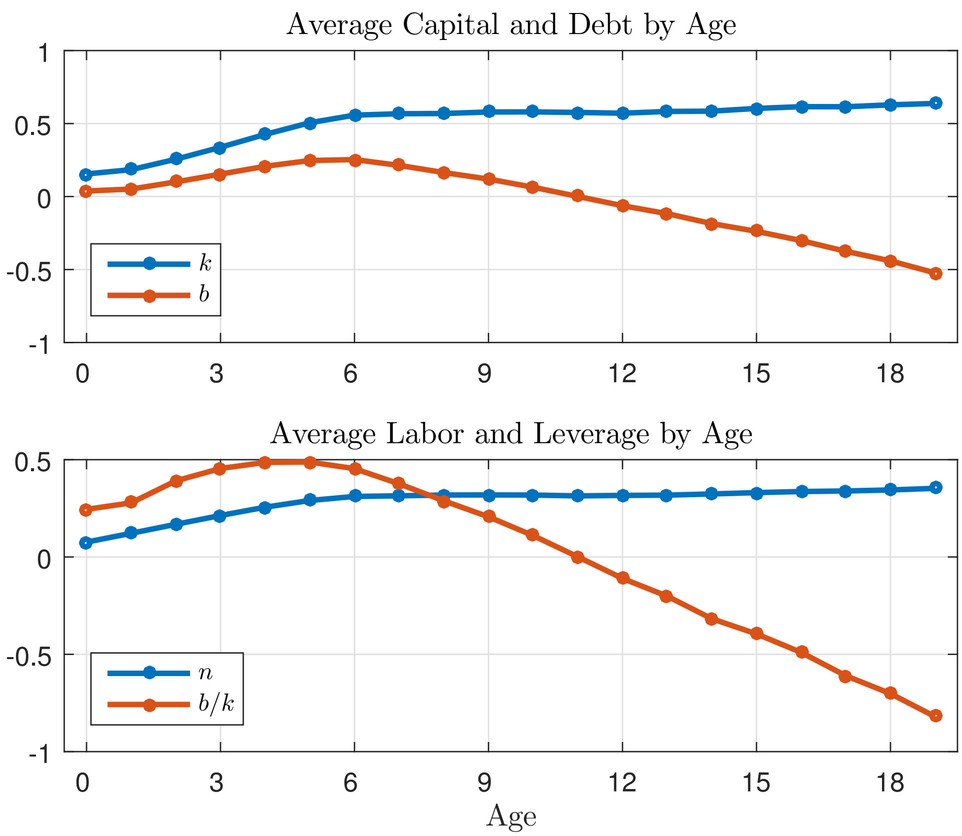

Figure 4 : Cohort average capital, debt or financial savings, labor and leverage ratio are constructed from a simulation of an unbalanced panel of firms.

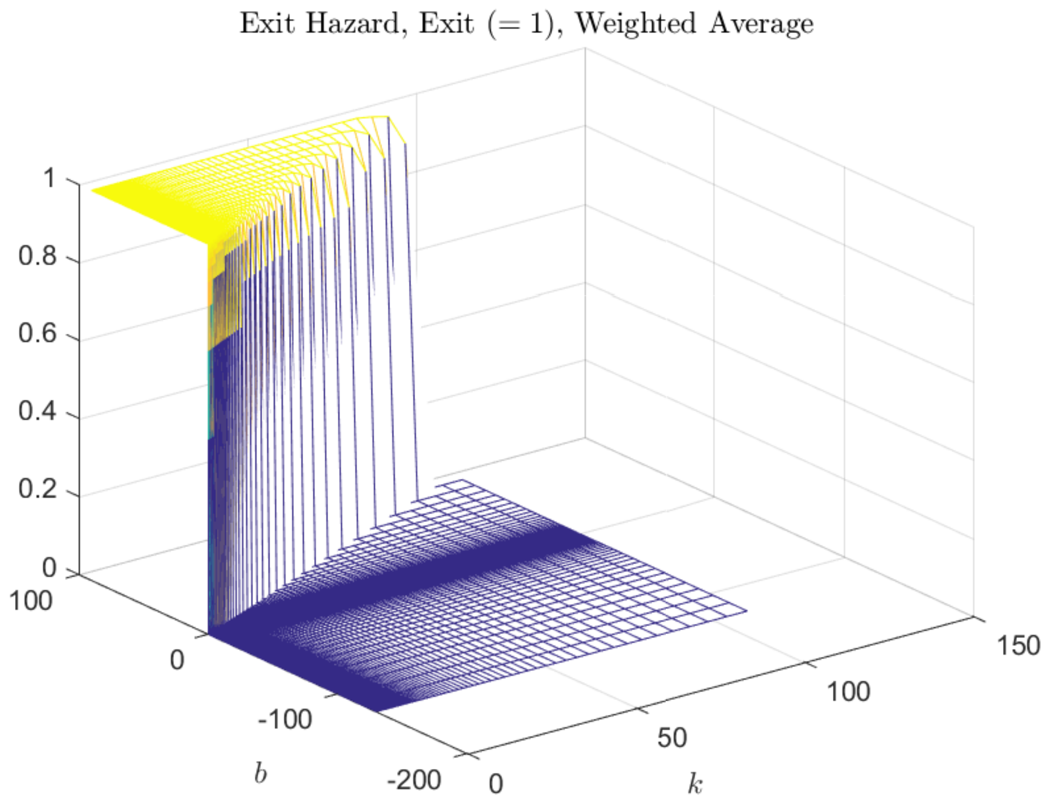

Figure 5 : Capital increases right to left. Debt increases front to back; negative values are financial savings.

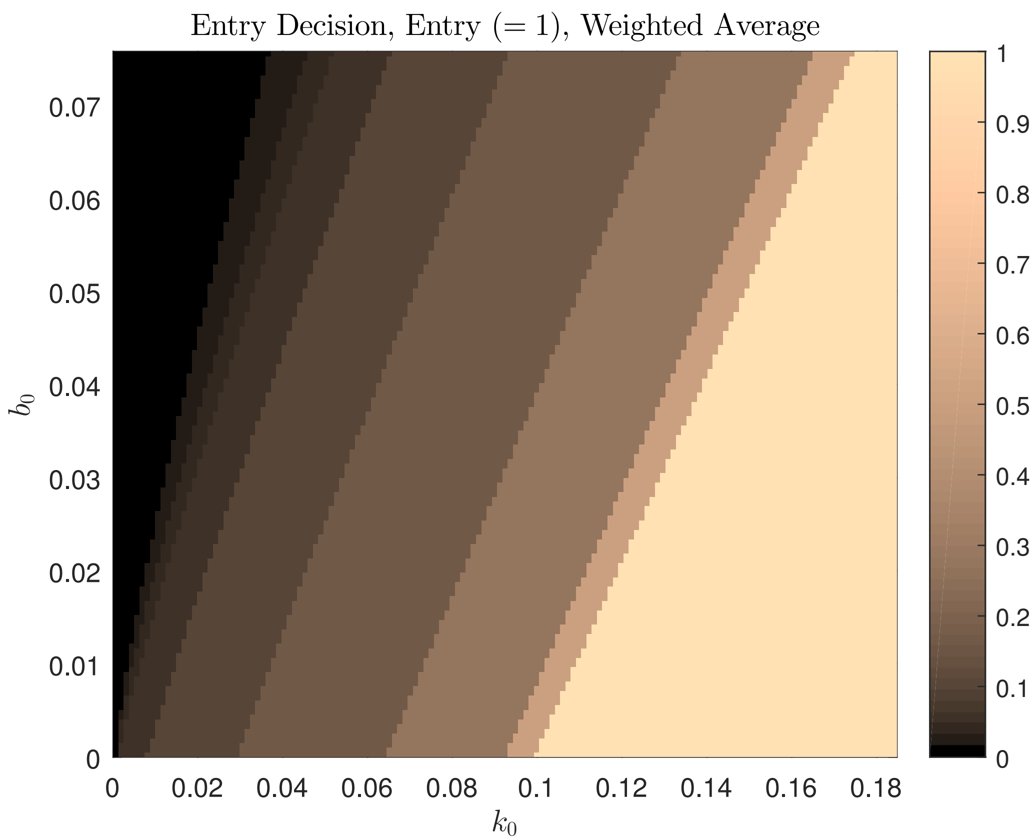

Figure 6 : Entry decision over the initial (uniform) distribution of capital and debt, (k_0, b_0), in the benchmark economy.

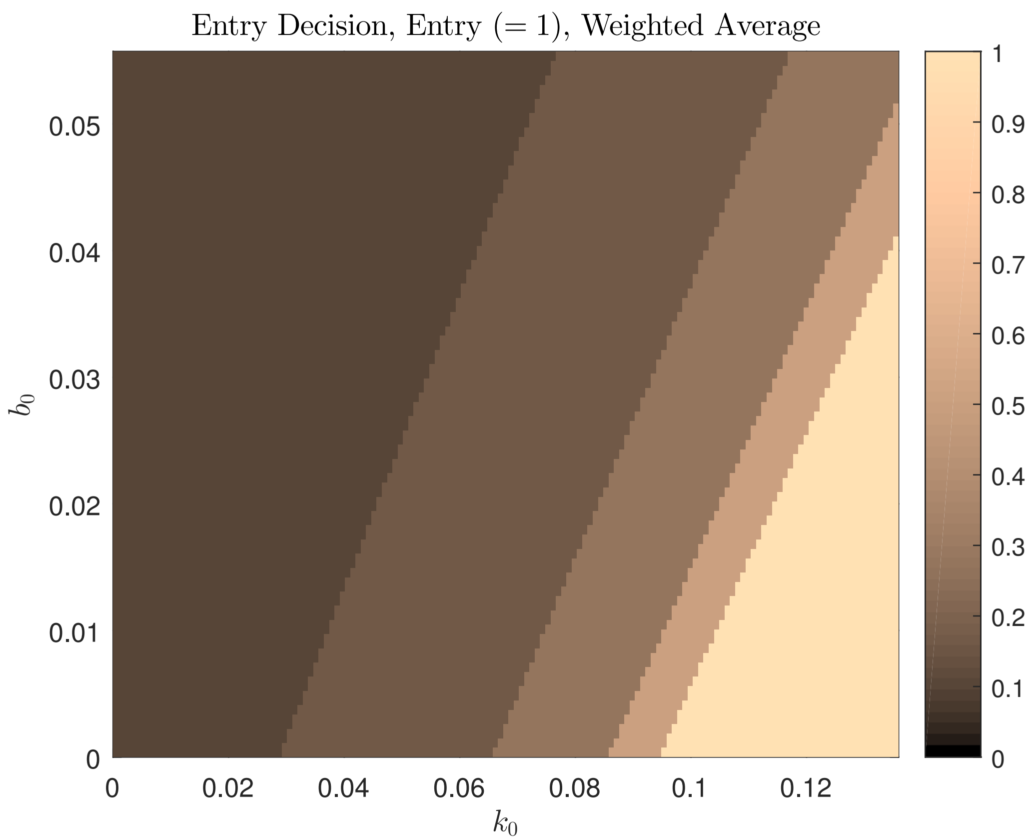

Figure 7 : Entry decision over the initial (uniform) distribution of capital and debt, (k_0, b_0), under the size-dependent policy.

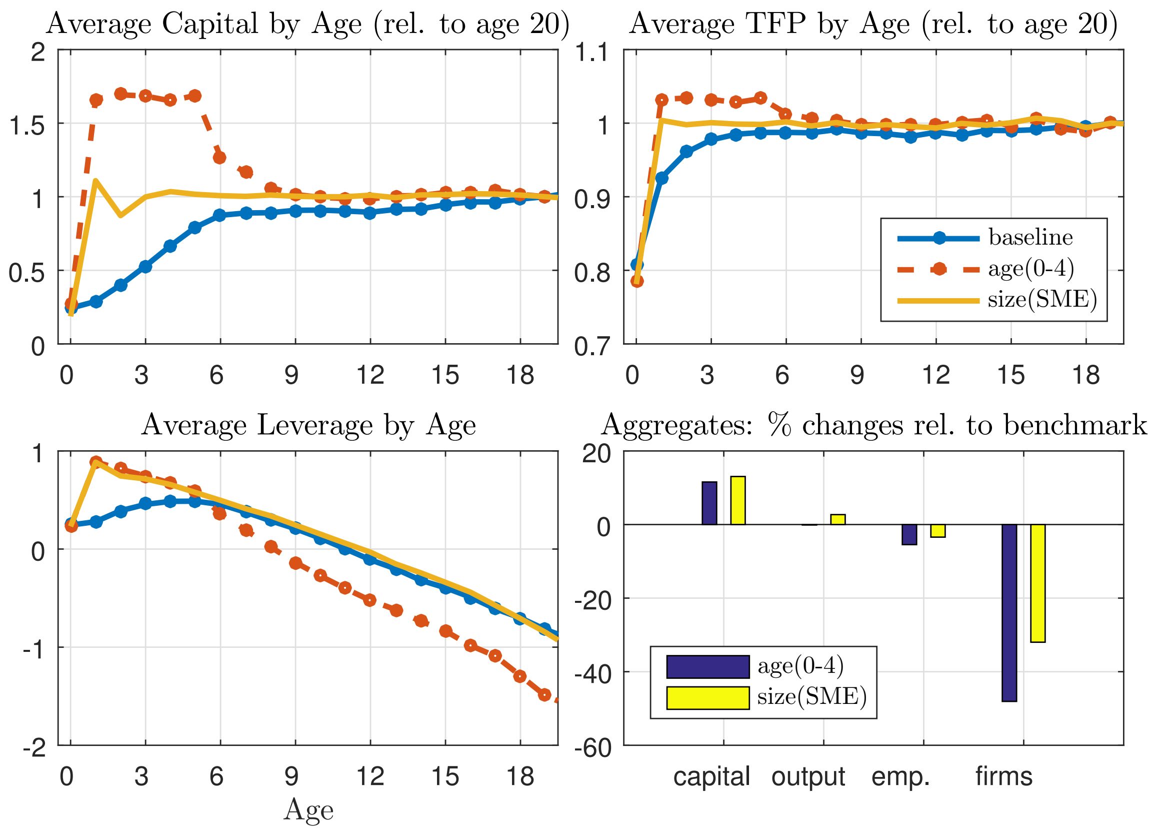

Figure 8 : Cohort average capital, TFP and leverage ratio are constructed from a simulation of an unbalanced panel of firms.

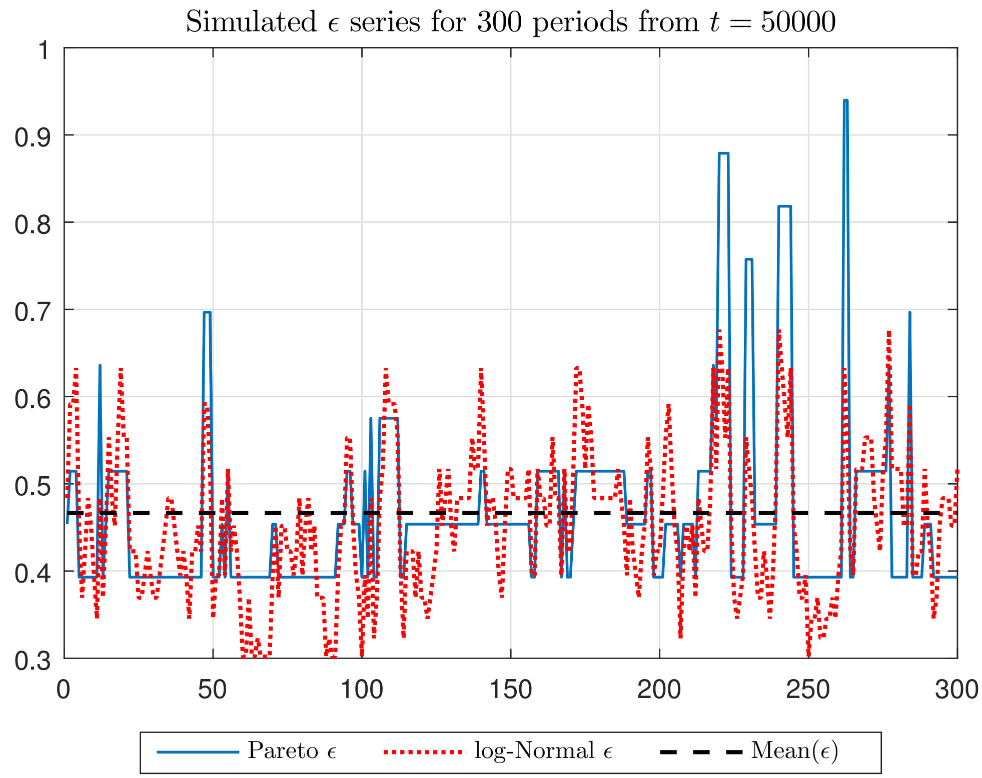

Figure 9 : Moments of the ergodic distribution for both Pareto and log-normal distributions are matched and the moments are: mean 0.466, standard deviation 0.112, coefficient of autocorrelation 0.750. Using the parameterized processes, a panel of firms are simulated and represented simulation paths are plotted in the above figure.

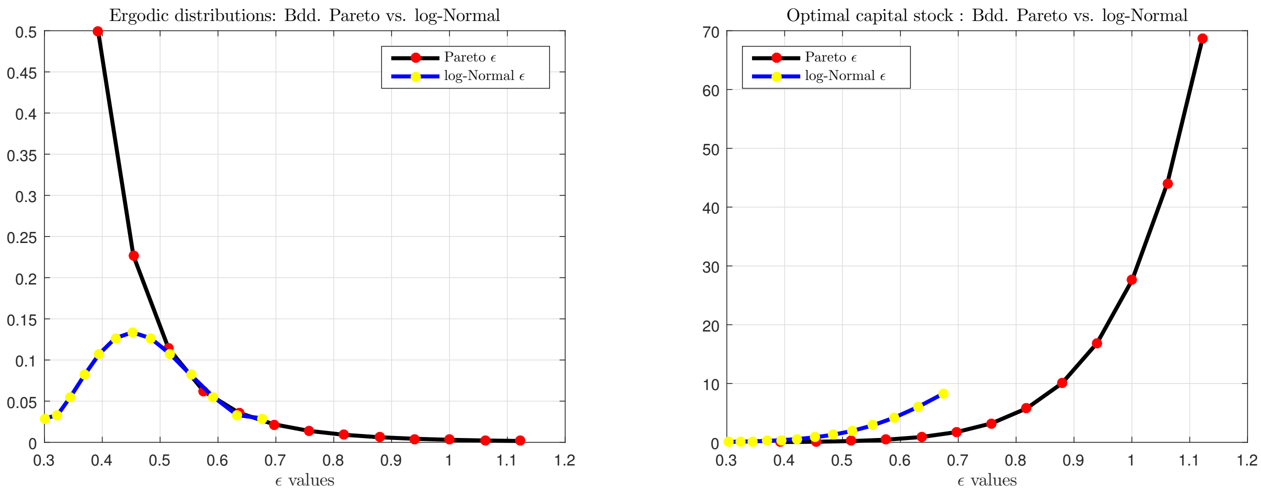

Figure 10 : The mass of firms is plotted for each of 13 productivity grid points on the left panel. The efficient capital choice is plotted against each of 13 productivity grid points on the right panel.

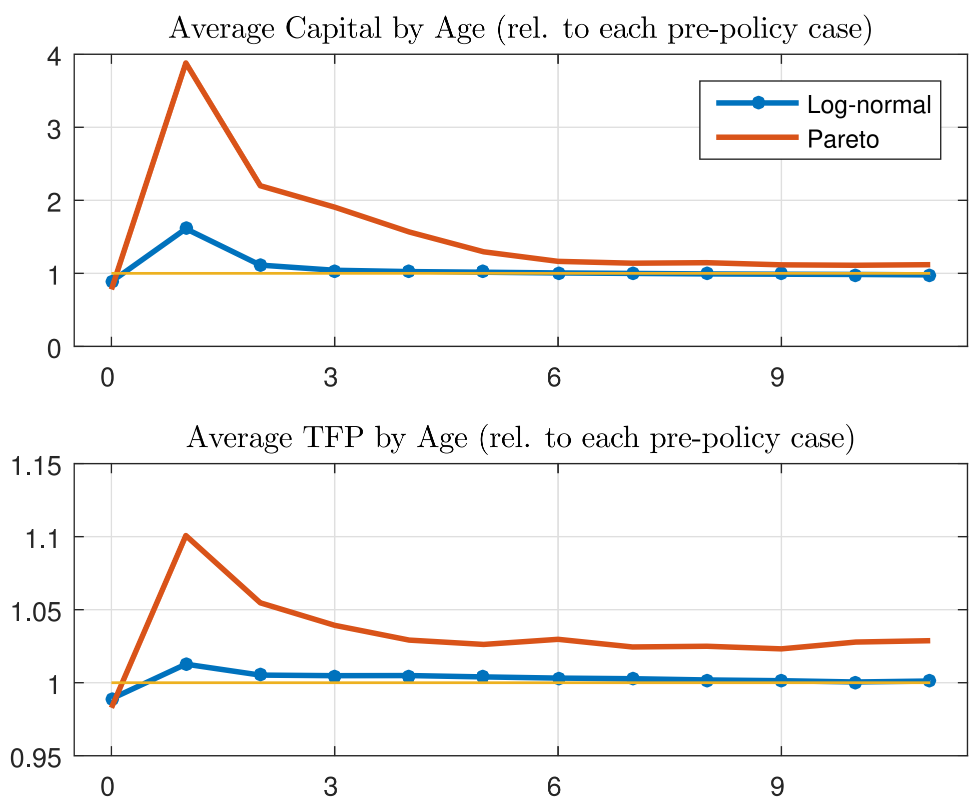

Figure 11 : Cohort average capital and TFP are constructed from a simulation of an unbalanced panel of firms.